Cost minimization, profit maximization & duality in a Faustmann framework

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Purpose

To describe and analyze the impact on harvesting and silvicultural decisions if forest land owners minimize costs rather than maximize NPV

Technical (boring) but important topic

Outline

- Justification
- Summary and Implications
- Analysis
  - Faustmann NPV functions and conditions
  - Cost minimization functions and conditions
  - Equivalence of NPV and cost minimization conditions
- Use of Cost Minimization Results
- Selected Comparative Statics Results

Justification: Landowner Behavior

- Private landowner behavior may be the most studied topic in forest economics

- Policy initiatives are often aimed at encouraging private land owners either to produce more timber or to produce more non-timber amenity services through an increase in silvicultural investments

- Studies are not particularly successful (first world); still much to learn

Justification: Cost Minimization I

- Minimizing costs is a simpler process than maximizing profits
  - Step 1 of 2 for profit maximization
  - Easier to analyze

- Duality is useful in economics

Justification: Cost Minimization II

Cost minimization may be more important in forestry than in other production processes

- Long rotation are in temporal & boreal forests
- Most production costs are incurred early in a rotation while most revenues and benefits are realized much later
- Often, the owner who plants a rotation will not be the owner harvesting it
- Cost information is more available and known; revenue information is often unavailable and uncertain

Justification: All Faustmann Models

- Infinite rotation, land sale & land rent models are equivalent if markets work (Samuelson 1976)
- Cost minimization needs to be developed or better developed for all models
- Creditable and useful analysis requires not conditions only analysis for all models, but it needs to be possible to move between the models as we can when maximizing NPV

What do landowners do?

- We do not know if landowners maximize the NPV of land or minimize costs (timber companies may maximize NPV)

- Landowners may:
  - Maximize profits (Faustmann NPV maximization)
  - Minimize costs valuing both silvicultural effort & land
  - Minimize explicit (silvicultural) costs and ignore implicit (land) costs
  - Minimize costs early in a rotation but maximize NPV later in rotation (original problem for us)

What do landowners do?

- If landowners minimize costs, then models developed and policies designed for landowners who maximize NPV may be ineffective

- We assume
  - Forest landowners produce a forest for harvest
  - Forest land eventually produces a forest of minimum size for harvest

Literature Review: Duality & Faustmann

- Desk drawers and computer files

- Brazee and Amacher (2000)
  - Strengths
    - Comparative statics results for taxes are correct
    - Provides rudimentary basis to analyze costs rather revenues in empirical studies
  - Weaknesses
    - Analysis is limited
    - Result is for the land sale model hybrid between infinite rotation and land sale model

Summary

New conditions are derived

- Generalize previous duality analysis of both the infinite rotation and land sale Faustmann models
- Extend duality analysis to land rent version of the Faustmann model
- Explicitly show links between models
- Include comparative statics for cost minimizing landowners

Selected Implications

- Comparative statics results for cost-minimizing landowners are more likely to be unambiguous than for the profit maximizing landowners

- Enhances our understanding of landowner behavior

- Provides a firmer foundation basis on which to analyze costs rather revenues in empirical studies

- Provides another explanation regarding why landowners may appear to under invest in forest management activities (Our original problem)

Infinite Rotation NPV Objective

Objective Function

$$Maximize\ VI(E,T) = \frac{e^{-rT}pQ(E,T) - wE}{1 - e^{-rT}}$$

$$w.r.t.\ E,T$$

where

VI(E,T) is the NPV of bareland

E is silvicultural effort

T is rotation age

Q(E,T) stumpage volume

p is stumpage price

r is the discount rate

w is marginal effort cost

Land Sale NPV Objective

Objective Function

$$Maximize\ VA(E,T) = \quad e^{-rT}(pQ(E,T) + A) - wE$$
$$w.r.t.\ E,T$$

where

VA(E,T) is the NPV of bareland, and

A is sale price of land

Land Rent NPV Objective

Objective Function

$$Maximize\ VZ(E,T) = \quad e^{-rT}pQ(E,T) - wE - \int_0^T e^{-rT} Z ds$$
$$w.r.t.\ E,T$$

where

VZ(E,T) is the NPV of bareland, and

Z is instantaneous land rent

Standard Faustmann Assumptions

- Constant parameters: A, p, r, w, Z
- Deterministic volume function Q(E,T)
- Start with bare land

Properties of Profit Functions

- Form a profit function (maximum profits as a function of parameters)
- Faustmann NPV functions meet required properties of profit functions (Varian 1994)
  - Non-decreasing in price; non-increasing in costs
  - Homogenous of degree one in price and costs
  - Convex in price and costs
  - Continuous in price and costs
- Faustmann NPV functions are profit functions

Imagine

Derivation of FOCs and SOCs for silvicultural effort and rotation age for each Faustmann NPV function

Cost Minimization: Land Sale

The cost function is:

$$CA\big(p, r, w, Q(.)\big) = \underset{\mathrm{w.r.t.\,E,T}}{\mathrm{Minimize}} \quad wE + A(1 - e^{-rT}) - p\bar{Q}e^{-rT}$$

Subject to a harvest volume constraint: $\bar{\mathrm{Q}} - \mathrm{Q}(\mathrm{E}, \mathrm{T}) = 0$

$\bar{\mathrm{Q}}$ is the required timber volume at harvest

Trick#1: Include interest forgone after harvest $-p\bar{Q}e^{-rT}$

Background on Trick

- Need to account for the cost of the interest forgone if harvest is delayed

- Costs may be included as $p\overline{Q}(1-e^{-rT})$

- But need to subtract $p\overline{Q}$

Constraint

- Simple output constraint follows Varian
- Could consider other constraints and get similar result
- Other constraints might include age, product quality, combination age and volume

Properties of Cost Functions

- Form a cost function (minimum costs as a function of parameters)
- Cost functions presented meet required properties of cost functions (Varian 1994)
  - Non-decreasing in costs (and price)
  - Homogenous of degree one in costs (and price)
  - Convex in costs (and price)
  - Continuous in costs (and price)
- Cost functions presented are cost functions

Cost Minimization: Land Sale

Set up a Lagrangian:

$$LA = wE + A(1 - e^{-rT}) - p\bar{Q}e^{-rT} + \lambda(\bar{Q} - Q(E,T))$$

Set the relevant derivatives equal to 0:

$$\mathrm{L}A_{\mathrm{E}} = w - \lambda Q_E(E,T) = 0;$$

$$\mathrm{L}A_{\mathrm{T}} = e^{-rT}(rA + rp\bar{Q}) - \lambda Q_T(E,T) = 0;$$

$$\mathrm{L}A_{\lambda} = \bar{Q} - Q(E,T) = 0;$$

Cost Minimization: Land Sale

As required the conditions that describe minimizing costs are equivalent to the conditions for maximizing NPV when λ and $\overline{Q}$ are set at the right levels

- First set discounted stumpage price equal to the multiplier
- Next set the output constraint at the price maximizing level of output

Cost Minimization: Land Rent

The cost function is:

$$CZ\big(p,r,w,Q(.)\big) = \underset{w.r.t.E,T}{Minimize}\; wE + \int_0^T e^{-rs} Z ds - p\bar{Q}e^{-rT}$$

Integrating provides:

$$CZ\big(p,r,w,Q(.)\big) = \underset{w.r.t.E,T}{Minimize}\; wE + \frac{Z}{r}(1 - e^{-rT}) - p\bar{Q}e^{-rT}$$

If land markets are perfect , then $\frac{Z}{r} = A$ (Samuelson 1976), and

$$CZ\big(p,r,w,Q(.)\big) = CA(p,r,w,Q(.))$$

$$= \underset{w.r.t.E,T}{\text{Minimize}}\quad wE + A(1 - e^{-rT}) - p\bar{Q}e^{-rT}$$

Cost Minimization: Land Rent

- Similar to the Land Sale case
- Imagine the derivation

Cost Minimization: Infinite Rotations

The cost function for the infinite rotation model can be derived from the cost function for the land sale model.

The cost function for the land sale model is:

$$CA\big(p, r, w, Q(.)\big) = \underset{\text{w.r.t. E, T}}{\text{Minimize}} \quad wE + A(1 - e^{-rT}) - p\bar{Q}e^{-rT}$$

Trick #2: Noting that the cost function of the asset sale model considers a single rotation, it can be converted to the infinite rotation by dividing through by 1-e^{-rT}:

$$CI\big(p, r, w, Q(.)\big) = \underset{w.r.t.E,T}{Minimize} \frac{wE}{1 - e^{-rT}} + \frac{A(1 - e^{-rT})}{1 - e^{-rT}} - \frac{p\bar{Q}e^{-rT}}{1 - e^{-rT}}$$

Cost Minimization: Infinite Rotations

Which reduces to:

$$CI\big(p, r, w, Q(.)\big) = \underset{w.r.t.\,E,T}{Minimize} \; \frac{wE - p\bar{Q}e^{-rT}}{1 - e^{-rT}} + A$$

Note that costs include the price of land but since the land is devoted to forestry in perpetuity, the land price is a fixed cost and does not impact rotation age

Cost Minimization: Infinite Rotations

Set up a Lagrangian:

$$LI = \frac{wE - p\bar{Q}e^{-rT}}{1 - e^{-rT}} + A + \lambda \frac{\bar{Q} - Q(E,T)}{1 - e^{-rT}}$$

Trick #3: Dividing the required harvest volume constraint by 1-e^{-rT} is correct since the constraint must be satisfied in every rotation

Cost Minimization: Infinite Rotations

- Solve similar to Land Sale Case
- Show that under the right assumptions that the conditions are identical to the Faustmann conditions
- Imagine

Importance of Equivalent Conditions

The previous argument is not made in Brazee and Amacher (2000)

By developing the analysis for all of the Faustmann Models (Land Sale, Land Rent and Infinite Rotation) and showing the connections between the models the current derivations are complete and form a basis for use

Use of cost minimization results

- Comparative statics help with policy design (e.g. stumpage prices, silvicultural effort, taxes)
- More easily frames problems in which some costs are ignored or undervalued
- Allows us to model landowners switching between cost minimization and NPV maximization
- Simulate underinvestment due to both cost minimization or a switch from cost minimization to NPV maximization (our initial problem)

Selected Infinite Rotation Comparative Statics Results Faustmann NPV Maximum

	E*	T*
Stumpage Price	+/-	+/-
Marginal Effort Cost	+/-	+/-
Interest Rate	+/-	+/-

Selected Infinite Rotation Comparative Statics Results Cost Minimization

	E*	T*
Stumpage Price	+	-
Marginal Effort Cost	-	+
Interest Rate	+	-

Why are Faustmann comparative statics results often unknown?

The matrix of second order partial derivatives is:

$$J = \begin{pmatrix} \frac{\partial^2 \mathrm{V}}{\partial \mathrm{E}^2} & \frac{\partial^2 \mathrm{V}}{\partial \mathrm{E} \partial T} \\ \frac{\partial^2 \mathrm{V}}{\partial \mathrm{E} \partial T} & \frac{\partial^2 \mathrm{V}}{\partial \mathrm{T}^2} \end{pmatrix}$$

The signs of the derivative are:

$$J = \begin{pmatrix} - & ? \\ ? & - \end{pmatrix}$$

Why are cost minimizing comparative statics results often known?

The matrix of second order partial derivatives is:

$$J = \begin{pmatrix} \frac{\partial^2 L}{\partial E^2} & \frac{\partial^2 L}{\partial E \partial T} & \frac{\partial^2 L}{\partial E \partial \lambda} \\ \frac{\partial^2 L}{\partial E \partial T} & \frac{\partial^2 L}{\partial T^2} & \frac{\partial^2 L}{\partial T \partial \lambda} \\ \frac{\partial^2 L}{\partial E \partial \lambda} & \frac{\partial^2 L}{\partial T \partial \lambda} & 0 \end{pmatrix}$$

The signs of the derivatives are:

$$J = \begin{pmatrix} + & ? & - \\ ? & + & - \\ - & - & 0 \end{pmatrix}$$