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Uncertainty, Extreme Outcomes and Climate Change: a critique

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Abstract

Building upon the work of Pindyck(2012), I show how different assumptions regarding the utility and damage functions can support the immediate adoption of a stringent abatement policy. I employ an additive rather than a multiplicative form for the utility function and a damage function that accounts for extreme climate change. Using the distribution for temperature change and the economic impact provided by Pindyck (2012), based on information from the IPCC (2007) and recent IAMs, I estimate a simple measure of "willingness to pay". My specifications lead to significantly higher estimations for the WTP than in Pindyck and in some extreme cases to a value close to 1. Although one could not strongly argue which is the right specification for the model, the analysis suggests that seemingly small differences in modelling can have very different policy implications.

1 Introduction

The issue of increasing greenhouse gas emissions and the potential impacts on welfare has been the subject of a growing literature which aims to join climate to economy, predict future temperature change and offer an insight regarding the optimal climate change policy: when, where and by how much to abate emissions. Most economic analyses of climate change policy have five elements: projections of CO_{2e} under a " business usual', projections of the average

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temperature change likely to result from higher CO_{2e} concentrations, projections of lost GDP and consumption resulting from higher temperatures, estimates of abatement cost of GHG emissions and assumptions about social utility and other ethical parameters. This is essentially the approach of Nordhaus [20], Stern [31], Hope [14] and others that evaluate abatement policies using "Integrated Assessment Models" (IAMs). Surprisingly, some of these models conclude -in direct contradiction of the urgency expressed in the scientific literature- that stringent abatement is both economically unsound and unnecessary.

The scope of this paper is to show how the deep structural uncertainties inherent in climate change economics can lead to very different implications for the optimal climate change policy. In Pindyck [25], the author concludes, that " given the current state of knowledge regarding warming and its impact, my results do not support the immediate adoption of stringent abatement policy." My analysis builds up on his work and aims to show how different assumptions about the utility function, the damage function and the distributions of the unknown parameters can support the immediate adoption of an abatement policy. Following Pindyck, I estimate a simple measure of the "willingness to pay": the fraction of consumption, $w^*(\tau)$ that society would be willing to sacrifice today and throughout the future to ensure that temperature change at some horizon H would be limited to τ. The value of $w^*(\tau)$ will depend on the distributions of temperature change and the economic impact as well as the utility function assumed and the values for the ethical parameters. It should be noted here that I am not considering whether the fraction of concumption sacrificed is enough to keep temperature change below τ but instead I focus entirely on how the inherent uncertainties about temperature change and economic impact would affect the WTP.

I use the distribution for temperature change and the economic impact provided by Pindyck [25] which is based on information from the IPCC (2007) and recent IAMs. As Weitzman [38] has shown, WTP becomes infinite if one uses a fat tailed distribution, therefore I follow Pindyck in using a thin tailed distribution for temperature change and economic impact. Unlike existing IAMs , he also assumes that temperature change affects the growth rate of GDP and consumption rather than the level. Although justified on theoritical and empirical grounds as Dell and Jones [8] have shown, I argue that the way he translates the economic impact in levels to one in growth is not so "innocent" and could be a source of the low WTP that he finds.

Pindyck uses a quadratic exponential loss function to relate temperature change and GDP. The choice of an exponential rather an inverse quadratic polynomial loss function (as the DICE damage function) allows for greater losses when ΔT is large. However, as have pointed out, there is no empirical evidence regarding the shape of the damage function, even at historical temperatures-let alone “out of the sample” forecasting of damages at temperatures beyond the historical range, which is what really matters in the context of uncertainty and extreme outcomes. In other words, there is no obvious source of support for the crucial assumption that the exponent is equal to 2. They use the well-established DICE damage function and study the effects of $\Delta T = 19^{\circ}C$ on the level of GDP for increasing values of the exponent N. They show that with $N = 2$, less than half of world output is lost although this temperature change is far beyond the temperature change needed to end human life as we know it. In contrast, as N rises, half of the world output is lost at $\Delta T = 7^{\circ}C$ for$N = 3$ and at $\Delta T = 4.5^{\circ}C$ for $N = 4$. As N approaches infinity, the damage function approaches a vertical line, which makes sense if one accepts that there is a threshold for an abrupt word ending (or at least economy ending) discontinuity. As I want to study the effects of extreme climate change, I will adopt the above approach and set $N = 3$.

The final matter of concern is the choice of the utility function of the individual. Pindyck, as well as most of the IAMs, uses a CRRA “multiplicative " functional form utility which implicitly assumes perfect substitutability between material consumption and environmental amenities. However, as Tol et al [10] have pointed out, the individual derives utility from environmental amenities, not necessarily translated to market consumption. As Weitzman [38] has pointed out, the choice of an additive rather than a multiplicative form for the utility function accounts for this fact and leads to significantly different implications about climate change policy when consumption and temperature change is high, with the additive form resulting to larger values for WTP.

Therefore, I employ an “additive" as well as a “multiplicative" form for the utility function and the corresponding damage function applied to the level of consumption rather than growth while allowing for an exponent of $N = 2$ and $N = 3$. The first immediate results are higher levels of damages for large temperature change that could represent extreme climate change. These results are along the same lines with the results of Ackerman [1] and Weitzman [38] who emphasize the importance of uncertainty regarding the shape of the damage function and the climate sensitivity parameter for the inclusion of potential catastrophes in IAMs and the justification of an immediate and stringent abatement policy. Given the current “ state of knowledge” Pindyck obtains estimates for the WTP which are generally below 3% even for τ around to $2-3^{\circ}C$. As he states, this is because there is limited weight in the tails of the calibrated distributions for ΔT and the growth rate impact. Instead, the specifications of the model I consider, lead to significantly higher estimations for the WTP and in some extreme cases to a value close to 1. The results are even stronger in the case of a larger excepted temperature change where even a quadratic exponential loss function applied to the level of consumption gives a higher WTP than the benchmark specification of Pindyck. Although Dell, Jones and Olken [8] have shown that higher temperatures reduce GDP growth rates rather than levels, the previous results cast doubts as to whether the specification of the model where temperature change affects the growth rather than the level of consumption brings robust estimates for the WTP. However, as it is hard to argue which is the “right" functional form, the discussion is aimed to point out that a seemingly arcane theoretical distinction between a “multiplicative" and an “additive" functional form can have very different implications for the optimal climate change policy.

The rest of the paper is organized as follows. Section 2 is dedicated to the methodology followed regarding the treatment of temperature change and its distribution, and the functional forms for the damage and utility functions . In Section 3, I describe the methodology followed for the estimation of WTP as well as the differences in the estimations resulting from the different specifications of the model. Section 4 concludes with the policy implications of my analysis.

2 Background and Methodology

Most economic analyses of climate change policy have five elements: projections of CO_{2e} under a “ business usual”, projections of the average temperature change likely to result from higher CO_{2e} concentrations, projections of lost GDP and consumption resulting from higher temperatures, estimates of abatement cost of GHG emissions and assumptions about social utility and other ethical parameters. In this section, I offer a descriptive analysis of the above. As Pindyck is the benchmark model for my analysis, there is an analytical description of his methodology regarding temperature change and economic impart as well as the possible issues that can arise.

2.1 Temperature Change

Following Pindyck [25], I use the estimates of the IPCC (2007a) for “climate sensitivity": the temperature change that would result from an anthropomorphic doubling of CO2e concentration. Climate sensitivity is then used as a proxy of temperature change a century from now, given the IPCC’s consensus prediction of a doubling (relative to the preindustrial level) of CO2e concentration by the end of the century. According to the 22 studies that the IPCC surveyed, it is found that temperature change has an expected value of 2.5° to 3.0°C.

I use the estimates of Pindyck [25] that with 17% probability, a doubling of CO2e would lead to a mean temperature increase of 4.5°C or more, with 5% probability to a temperature increase of 7.0 or more and with 1% probability to a temperature increase of 10.0°C or more. The 1% and 5% tails of the distribution for ΔT clearly represent extreme outcomes as temperature changes of such a magnitude are outside the range human experience.

I assume that temperature change follows a displaced gamma distribution and I fit it to the above summary numbers. Let θ be the displacement parameter, the distribution is given by :

$$f(x; r, \lambda, \theta) = \frac{\lambda^r}{\Gamma(r)}(x-\theta)^{r-1}e^{-\lambda(x-\theta)}, \qquad x \geqslant \theta \tag{1}$$

$$F(x; r, \lambda, \theta) = 1 - \frac{\Gamma(r, (x-\theta)\lambda)}{\Gamma(r)} \tag{2}$$

where

$$\Gamma(r) = \int_0^\infty s^{r-1}e^{-s}ds \tag{3}$$

is the Gamma function, r is the shape parameter and λ the inverse scale parameter.

The moment generating function for this distribution is:

$$M_{x(t)} = E\left(e^{tx}\right) = \left(\frac{\lambda}{\lambda - t}\right)^r e^{t\theta} \tag{4}$$

Hence, the mean, the variance and the skewness (around the mean) are given by $E(x) = \frac{r}{\lambda} + \theta$, $V(x) = \frac{r}{\lambda^2}$ and $S(x) = \frac{2r}{\lambda^3}$ respectively.

The next step is to fit $f(x; r, \lambda, \theta)$ to a mean of 3.0°C and the 1% and 5% points at 7.0° and 10.0°C respectively yields $r = 3.8$, $\lambda = 0.92$ and $\theta = -1.13$, with $V = 4.49$ and $S = 9.76$. Note that this distribution implies that there is a small probability of 2.9% that a doubling of CO2e concentration will lead to a reduction in temperature change which is consistent with several of the scientific studies. These are the estimates given by Pindyck [25]. As in Pindyck, I assume that the fitted distribution for T applies to a 100-year horizon H and that $\Delta T_t \to 2\Delta T_H$ as t gets large. This implies that the trajectory for temperature change is given by

$$\Delta T_t = 2\Delta T_H(1 - 0.5^{\frac{t}{H}}) \tag{5}$$

Therefore, if $\Delta T_H = 5$°C ,ΔT reaches 2.93°C after 50 years, 5°C after 100 years and then gradually approaches 10°C as t gets large.

2.2 Economic Impact and Choice of Functional Forms

It has become common in the literature to assume that temperature change and GDP are associated through a loss function $L(\Delta T_t)$, with $L(0) = 1$ and $L^{'} < 0$ so that GDP at some horizon H is $L(\Delta T_H)\, GDP_H$, where GDP_H is GDP in the absence of global warming. Many of the IAMs use a simple power law or an inverse quadratic loss function as in the DICE model of Nordhaus [20] while Weitzman [39] argues for an exponential loss function which allows for greater losses when temperature change is large. On the other hand, there are reasons to believe that temperature change would affect the growth rate of GDP rather than just the level. One could think of irreversible and permanent effects of climate change as for example the destruction of ecosystems from erosion and flooding, extinction of species and deaths from health effects and weather extremes. Second, climate change leads to a reduction of resources available for R&D and capital investment and therefore reduces growth as it has a negative impact on the level of GDP and also because of the resources that need to be allocated to counter the floods, droughts, sickness, etc resulting from higher temperatures. Third, there is the effect of climate change on savings. As Frankhauser et al. [10] point out, in a world with perfect foresight we can expect forward- looking agents to change their savings behavior in anticipation of future climate change. This, too, will affect the accumulation of capital and hence growth and future GDP. Finally, there is also empirical support for a growth rate effect. Dell, Jones and Olken [8] have shown that higher temperatures reduce GDP growth rates rather than levels. The impact they estimate is large-a decrease of 1.1 percentage point of growth for each 1°C but significant only for poorer countries. In the benchmark model of my analysis, Pindyck assumes that in the absence of warming, real GDP and consumption would grow at a constant rate g_0 but warming will reduce this rate :

$$g_t = g_0 - \gamma \Delta T_t \tag{6}$$

This simple linear relationship was estimated by Dell, Jones and Olken (2008), fits the data well and can be viewed as at least a first approximation to a more complex function. Assuming that the loss function applied to the levels has the following functional form

$$L(\Delta T_t) = e^{-\beta (\Delta T_t)^2} \tag{7}$$

Pindyck uses information from a number of IAMs to obtain a distribution for β and then translate this into a distribution for γ. To do this translation, he uses the trajectory for GDP and consumption implied by equation (5) for a temperature change-impact combination projected to occur at horizon H, so that the growth rate is given by

$$g_t = g_0 - 2\gamma\Delta T_H(1 - 0.5^{t/H}) \tag{8}$$

Normalizing initial consumption at 1, consumption at any time t is given by the following expression

$$C_t = e^{\int_0^t g(s)ds} \tag{9}$$

By substituting for the growth rate according to (8) and solving for the integral, (9) becomes

$$C_t = e^{\int_0^t g(s)ds} = e^{(g_0 - 2\gamma\Delta T_H)t - \frac{2\gamma H\Delta T_H}{\ln 0.5}(1-0.5^{t/H})} \tag{10}$$

Noting that consumption at any time t should be the same whether I assume that temperature change affects the levels or the growth rate of GDP/consumption, γ can be obtained by equating the expressions of C_H implied by (7) and (8):

$$e^{g_0 H - \beta(\Delta T)^2} = e^{(g_0 - 2\gamma\Delta T_H)H - \frac{2\gamma H\Delta T_H}{\ln 0.5}0.5} \tag{11}$$

so that

$$\gamma = 1.79\beta\Delta T_H/H \tag{12}$$

The next step is to estimate β and hence γ. Although there is no knowledge regarding the impact of extreme temperature change in the range of 7°C or above, there is a consensus regarding the most likely range of economic impacts corresponding to the most likely range of temperature change. Based on its own survey of impact estimates from four IAMs, the IPCC(2007b) concludes that global mean losses could be 1-5% fo GDP for 4.0C of warming.[1] In addition, Dietz and Stern [9] provide a graphical summary of damage estimates from several IAMs, which yield a range of 0.5% to 2% of lost GDP for $\Delta T = 3.0$°C and 1% to 8% of lost GDP for $\Delta T = 5.0$°C. Using the IPCC range and assuming that it applies to a 66% confidence interval, I take the mean loss for $\Delta T = 4.0$°C to be 3% of GDP and the 17% and 83% confidence points to be 1% and 5% of GDP respectively. From these 3 numbers, using the exponential loss function we can find the mean, the 17% and 83% values for β which are $\bar{\beta} = 0.0019037$, $\beta_1 = 0.000628$ and $\beta_2 = 0.0032$ respectively. Pidyck translates these values

[1] The IAMs surveyed by he IPCC include Hope [13], Mendelsohn et at [18], Nordhaus and Boyer [23] and Tol [33].

of β to values of γ through (12) and gets $\bar{\gamma} = 0.0001367$, $\gamma_1 = 0.000045$ and $\gamma_2 = 0.00023$. Assuming that the distributions of ΔT and γ are independent given that they are governed by completely different physical economic/processes, he then fits a 3- parameter displaced gamma distribution for γ using the above numbers and he gets $r_\gamma = 4.5$, $\lambda_\gamma = 21.341$ and $\theta_\gamma = -0.0000746$.

A closer look to the assumptions and methods followed above reveals some non-trivial weaknesses that ought to be discussed. The first issue is the choice of the damage function. As noted in the introduction, there is no obvious source of support for the crucial assumption of an exponential quadratic loss function as presented in (7). where the exponent N is set equal to 2.[2] As Ackerman et al. [1] have pointed out, the exponent N measures the speed with with which damages increase as temperature rises. Choosing a larger ("closer to infinity") N means moving closer to the view that complete catastrophe sets in at some finite temperature threshold. Choosing a smaller N means emphasizing the gradual rise of damages rather than the risk of discontinuous catastrophic change. As I am interested in the effects of uncertainty and extreme outcomes on the willingness to pay, I am going to use $N = 3$ as a specification of my model and compare it to the benchmark model with $N = 2$. The first obvious effect of using a larger value for N is the increase in the loss of GDP for high temperatures. For example, for $N = 3$ and an exponential loss function, I get $\bar{\beta} = 0.0006345$ and so for $\Delta T = 10°C$ (which is outside the range of human experience), almost half of the world output would be destroyed while $N = 2$ would lead to a destruction of only around 20% of world output. However, the difference is trivial for small temperature changes so for example for $\Delta T = 3.0°C$, the loss is 1.7% and 1.69% of GDP for $N = 2$ and $N = 3$ respectively.

Even if one chooses an exponent high enough to accommodate for the losses of extreme temperature change, there is an even more important source of inefficiency in the estimation of the economic impact of climate change on welfare: the functional form of the loss function, whether "multiplicative" or "additive", will have very different implications for the magnitude of the impact of temperature change. It has become common in the literature of climate change and IAM's to assume a multiplicative functional form for the loss function either with rational bounding or with exponential bounding given by

$$L_{MR}(\Delta T_t) = \frac{1}{1 + a_M \Delta T_t^2} \tag{13}$$

and

[2]For an exponential loss function, $L(\Delta T_t) = e^{-\beta(\Delta T_t)^N}$, the exponent N measures the speed with which damages increase as temperature rises.

$$L_{ME}(\Delta T_t) = e^{-\beta(\Delta T_t)^2} \quad (14)$$

respectively where the latter form gives larger losses for high temperature change.[3] Then, a CRRA utility function corresponding to the first multiplicative damage function is of the form

$$U_{MR}(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)}(1 + a_M \Delta T_t^2)^{\eta-1}}{1-\eta} \quad (15)$$

and for the second

$$U_{ME}(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)} e^{-\beta \Delta T_t^2 (1-\eta)}}{1-\eta} \quad (16)$$

where c_t^* is consumption in the absence of temperature change and η is the index of relative risk aversion.[4]

The above functional form assumes that there is strong substitutability between its two attributes, consumption and temperature change. It best fits situations where the main economic impact of temperature change is in material consumption or consumption of market goods. However, as Fankhauser and Tol [10] have pointed out, the individual can derive utility from non market goods, as for example environmental amenities. This idea is embodied in what is so called an "existence" value for environment. Hence, climate change which, in the model considered, is represented by a temperature change, has market impacts which go through the increase of production cost and affect the growth rate of the economy, but also non-market impacts such as the effect on recreational and environmental assets, health and biodiversity. In this account,Weitzman [39] shows that using a "additive" rather than a "multiplicative" functional form for the loss function and the corresponding utility function takes into account non-market impacts of climate change. He suggests the following "additive" loss function

and the implied "additive" utility function

$$L_A(\Delta T_t) = \frac{1}{1 + a_A c_t^* \Delta T_t^2} \quad (17)$$

[3] At it will be shown later, WTP is higher in the case of multiplicative damage function with exponential bounding than with rational bounding.

[4] It should be noted that $U(c_t^*, \Delta T_t)$ represents U only as a reduced form in c_t^* and ΔT_t. The indirect pathway via which temperature change affects welfare is through reducing actual consumption as represented by the loss function $c_t = L(\Delta T_t)c_t^*$ so that $V(L(\Delta T_t)c_t^*) = U(c_t^*, \Delta T_t)$

$$U_A(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)}(1 + a_A c_t^* \Delta T_t^2)^{\eta-1}}{1-\eta} \tag{18}$$

where it is implicitly assumed that there is no substitutability between market and non-market goods and consumption. Although it is hard to argue which is the “right" functional form, the discussion is aimed to point out that a seemingly arcane theoretical distinction between a “multiplicative" and an “additive" functional form can have very different implications for the optimal climate change policy. It will be shown later that, especially for high temperature change, willingness to pay to avoid climate change is much higher under an “additive" utility function.

The same discussion applies for the choice of function that relates growth to temperature change. Pindyck assumes a simple linear functional form, the same that Dell and Jones use in their analysis. Although it is a good approximation of a possibly more complicated relationship and fits the data well, it does not account for the effects of large temperature change. First, Pindyck argues that he uses an exponential loss function in order to allow for larger losses for high temperature change in the levels of consumption, which he then translates into an effect on the growth level. However, the choice of the loss function is irrelevant for the estimation of γ: only the estimated loss from a range of likely temperature change scenarios is needed for the estimation of the effect on growth. Therefore, the choice of the growth function becomes even more important as it will determine the magnitude of the impact of high temperature change and extreme events. To make this point clear, assume that $g_0 = 0.02$,$\gamma = 0.\bar{0}001367$, and $\Delta T_H = 20.0C^\circ$. Equation (8) then implies a decrease of 0.0003 in the growth rate which is tiny for a temperature change which is well beyond the temperature needed to cause the end of human life as we know it. It comes as no surprise then that the benchmark model concludes that even if people face the risk of an extreme climate change event that would end human life as we know it, willingness to pay for abatement technology would still be very low.

3 Willigness to Pay

The preferences of an agent are described by a CRRA utility function:

$$V(c_t) = \frac{c_t^{1-\eta}}{1-\eta} \tag{19}$$

where η is the index of relative risk aversion. As discussed earlier, $U(c_t^*, \Delta T_t)$ represents utility as a reduced form in c^* and ΔT so that

$$V(L(\Delta T_t)c_t^*) = U(c_t^*, \Delta T_t)$$

Time is continuous and we normalize population to be constant at N=1. Then, social welfare is:

$$W = E_0 \int_0^\infty U(c_t^*, \Delta T_t)\, e^{-\delta t} dt \tag{20}$$

where δ is the pure time preference rate and E_0 denotes the expectation at $t = 0$ over the distribution of the unknown parameters.[5] I assume that there is just one individual at each point in time (or a group of identical individuals). Alternatively, one could think of the problem as one of a representative consumer that lives for infinite time. That is to say that the lifetime well-being of a person is constructed in the same way as intergenerational well-being is constructed, which means that even if a person lives for many periods, she is considered to be a distinct self at each point in time. However, one could argue here that the decision about how much an individual would save for his children involves quite different ethics than the ones involved in the decision about how one should spread his consumption across time.

In the benchmark model with a linear function describing the effect of temperature change on growth, social welfare is given by:

$$W = \frac{1}{1-\eta} E_0 \int_0^\infty e^{-\rho t + \omega(1-0.5^{t/H})} dt \tag{21}$$

where $\rho = (\eta - 1)(g_0 - 2\gamma\Delta T_H) + \delta$ and $\omega = \frac{2(\eta-1)\gamma\Delta T_H}{\ln(0.5)}$

In order to study the implications of different functional forms for the loss and utility functions, I consider three different specifications for my model: a multiplicative damage function with exponential bounding, one with rational and an additive one. Taking into account the relevant functional forms for utility as given in (15), (16),(18) and noting that consumption in the absence of temperature change at time t is given by $c_t^* = e^{g_0 t}$, social welfare is given by

$$W_{ME} = \frac{1}{1-\eta} E_0 \int_0^\infty e^{-\delta t - (\eta-1)(g_0 t - 4\beta\Delta T_H^2 (1-0.5^{t/H})^2)} dt \tag{22}$$

[5] As discussed earlier, I have assumed a displaced gamma distribution for temperature change and for the parameter γ.

$$W_{MR} = \frac{1}{1-\eta} E_0 \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4\alpha_M \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{23}$$

$$W_A = \frac{1}{1-\eta} E_0 \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4e^{g_0 t} \alpha_A \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{24}$$

respectively.

In these three specifications, temperature change does not the enter the growth function of consumption. Instead, temperature change and consumption are associated through a loss function $L(\Delta T_t)$, with $L(0) = 1$ and $L' < 0$ so that consumption at some horizon H is $L(\Delta T_H) c_H^*$, where c_H^* is consumption in the absence of global warming. Note, that in order to avoid integrals that blow up, WTP must be based on some finite horizon, which I set to be N=500 years.

Suppose society sacrifices a fraction $w(\tau)$ of present and future consumption to ensure that any increase in temperature at a specific horizon H, is limited to an amount τ. This fraction is the measure of the willingness to pay. In this case, social welfare at $t = 0$ would be :

$$\tilde{W}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\tilde{\rho} t + \tilde{\omega}(1-0.5^{t/H})} dt \tag{25}$$

$$\tilde{W}_{ME}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - (\eta-1)(g_0 t - 4\beta \Delta T_H^2 (1-0.5^{t/H})^2)} dt \tag{26}$$

$$\tilde{W}_{MR}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4\alpha_M \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{27}$$

$$\tilde{W}_A(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4e^{g_0 t} \alpha_A \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{28}$$

where $E_{0,\tau}$ denotes the expectation at $t = 0$ over the distributions of ΔT_H and γ conditional on $\Delta T_H \leqslant \tau$.[6] (I use $\tilde{\omega}$ and $\tilde{\rho}$ to denote that they are both functions of random variables.) If on the other hand, no action is taken to limit warming, ΔT_H is unconstrained and social welfare is given by (21), (22), (23) and (24), Willingness to pay to ensure that $\Delta T_H \leqslant \tau$ is

[6]Note that I have assumed a displaced gamma distribution for temperature change and the parameter γ as given in (12).

the value $w^*(\tau)$ that equates $\tilde{W}(\tau)$ and W, social welfare without any action taken, for all the specifications of the model. By constraining ΔT_H to be under some threshold τ, damages from temperature change will also be constrained and a lower bound for consumption will be guaranteed. If no action is taken, the distribution temperature change is unconstrained and so are the damages and lost consumption. WTP is the fraction of consumption you could sacrifice today and in the future that makes you exactly indifferent between taking and not taking any action.

Given the distributions $f(\Delta T)$ and $h(\gamma)$ for ΔT and γ respectively, denote by $M_\tau(t)$, $M_\infty(t)$ the constrained and unconstrained time- t expectations for the benchmark model:

$$M_\tau(t) = \frac{1}{F(\tau)} \int_{\theta_\tau}^{\tau} \int_{\theta_\gamma}^{\infty} e^{-\tilde{\rho} t + \tilde{\omega}(1-0.5^{t/H})} f(\Delta T) g(\gamma) d\Delta T d\gamma \tag{29}$$

$$M_\infty(t) = \int_{\theta_\tau}^{\infty} \int_{\theta_\gamma}^{\infty} e^{-\tilde{\rho} t + \tilde{\omega}(1-0.5^{t/H})} f(\Delta T) g(\gamma) d\Delta T d\gamma \tag{30}$$

where θ_τ and θ_γ are lower limits on the distribution for ΔT and γ and $F(\tau) = \int_{\theta_\tau}^{\tau} f(\Delta T) d\Delta T$. The constrained expectation refers to the case when action is taken to limit temperature change and the unconstrained when no such action is taken. Therefore, social welfare in the case of action and no action taken, as given in (21) and (25), becomes

$$\tilde{W}(\tau) = \frac{[1-w(\tau)]^{1-\eta}}{1-\eta} \int_0^N M_\tau(t) dt = \frac{[1-w(\tau)]^{1-\eta}}{1-\eta} G_\tau \tag{31}$$

$$W = \frac{1}{1-\eta} \int_0^N M_\infty(t) dt = \frac{1}{1-\eta} G_\infty \tag{32}$$

where $G_\tau = \int_0^N M_\tau(t) dt$ and $G_\infty = \int_0^N M_\infty(t) dt$

FInally, setting $\tilde{W}(\tau)$ equal to W, WTP is given by :

$$w^*(\tau) = 1 - [G_\infty / G_\tau]^{1/1-\eta} \tag{33}$$

Following the same method, I can find the WTP for the three specifications of my model, namely $\tilde{w}^*_{ME}(\tau)$, $\tilde{w}^*_{MR}(\tau)$ and $\tilde{w}^*_{A}(\tau)$.

3.1 Parameter Values

Finally, one should discuss the values assumed for the "ethical" parameters, namely the rate of time preference (or the utility discount rate) δ and the consumption discount rate R. A good starting point is the so-called Ramsey rule

$$R_t = \delta + \eta g_t \tag{34}$$

where η is the index of relative risk aversion and g_t is the growth rate of consumption. The first component of, δ, implies discounting of future utility while the second implies discounting the value of future consumption simply because we will be richer in the future and the rich gain less welfare than the poor for a given quantity of money.

There are two approaches for the choice of the consumption discount rate. The prescriptive approach sets δ and η based on ethical views and then calculates R according to the Ramsey rule. In contrast, the descriptive approach sets R based on descriptions of the financial market interest rates and then calculates δ and η. Descriptivists are often flexible on the specific values of δ and η as long as they combine to give the desired level of R. As Dasgupta argues, descriptivism "is an interesting democratic work in that the values are generated by people's behavior as they go out in their daily lives. However, one should think here who are these people whose behavior we observe ". The market interest rates describe the behavior of current individuals only. One could ask the question why the interest of future generations should be determined by consulting the preferences of present generation (Baum, [3]), an approach which is often called "dictatorship of the present". Given that the decisions of the present generation will have a long run impact that will affect the welfare of the future generations, excluding future humans is violating the basic principle that those who are affected by a decision should have input in the decision-making process. One should also note here that, due to externalities and the absence of a market for environmental amenities, the social discount rate could be substantially different from the private rate of return. Finally, one should take into account the existence of multiple market rates so that analysts cannot consider any single market rate to describe how society discounts.

Hence, I stick with the prescriptive approach and I set $\delta = 0$ on the grounds that even though most people would value a benefit today more highly than a year from now, there is no reason why society should impose those preferences on the wellbeing of our grandchildren. Cline [5] also sets $\delta = 0$ while Stern sets $\delta = 0.01$ to account for the probability of extinction. However, they both find that stringent abatement policy is necessary. In contrast, Nordhaus [20] sets $\delta = 0.03$ and concludes that aggressive abatement is not optimal. Pindyck also assumes $\delta = 0$ and the consumption discount rate is given by

$$R_t = \delta + \eta g_t = \delta + \eta g_0 - 2\eta\gamma\Delta T_H(1 - 0.5^{\frac{t}{H}}) \tag{35}$$

Note that in this specification, R is endogenous: it depends on the growth rate of consumption g_t which is function of temperature change ΔT_t.

As for the index of relative risk aversion, I set $\eta = 2$, which is to say that individuals are very risk averse towards inequalities in income among generations (or in different periods of their lifetime). Based upon empirical evidence, g_0 is in the range of $0.02 - 0.25$. In my estimations, I will set $g_0 = 0.015 - 0.025$.

3.2 No uncertainty

If the trajectory for ΔT and the impact of that trajectory on economic growth and in the loss functions were all known with certainty, social welfare with and without action would simply be:

$$W_1 = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} \int_0^N e^{-\rho_\tau t + \omega_\tau (1 - 0.5^{t/H})} dt \tag{36}$$

$$W_2 = \frac{1}{1-\eta} \int_0^N e^{-\rho_0 t - \omega t^2} dt \tag{37}$$

where $\rho_\tau = (\eta - 1)(g_0 - 2\gamma\tau) + \delta$ and $\omega_\tau = \frac{2(\eta-1)\gamma\tau}{\ln(0.5)}$. By setting $\Delta T_H = \tau$, I find social welfare under no uncertainly for all the specifications of my model.

I start by calculating the WTP to keep $\Delta T = 0°C$ for all time, i.e. $w^*(0)$, over a range of values for ΔT at a horizon of 100 years. Here, I am using the mean $\bar{\gamma}$ as the certainty equivalent value of γ and I set N=500, $\eta = 2$, $\delta = 0$, and $g_0 = 0.015, 0.02, 0.025$ [7]. The results for the different specifications of the model and different combinations of the parameters are shown in Table 1. For example, in the benchmark specification, for $\Delta T_H = 6°C$ and $g_0 = 0.02$, $w(0)^* = 0.022$ which means that society would be willing to sacrifice about 2.2% of current and future consumption to keep temperature change at 0°C instead of 6°C.

The first thing to notice is that the additive specification gives a value of WTP much higher than every other specification: even for small expected temperature change, such as $\Delta T = 2°C$, WTP is in the range of 8%-14%, a range much higher than the one implied by the

[7] Similarly, I use the mean value of β, α_A and α_M for the other specifications of my model.

Pindyck model and by the alternative multiplicative specifications. Therefore, it appears that whether we assume that market goods and non-market goods are substitutable or not, has a determining effect on the optimal climate change policy. One could argue that, climate change impacts of a specific nature such as health and biodiversity qualify as non-market goods which cannot be readily translated to material consumption. In that case, perhaps an additive utility function is more appropriate from a theoretical perspective while accounting for the severity of large temperature changes and the urgency of action that the environmental scientists seem to prescribe.

A few more points worth to be noted. First, an exponent of 3 or larger in the damage function, increases WTP significantly for every specification, especially for larger changes in temperature. This result comes as no surprise: allowing for a larger exponent emphasizes the effects of extreme climate change on consumption and consequently on the willingness to pay. Comparing the case of growth effect of temperature with the one on the levels, WTP is similar for small temperature change but becomes much higher in the latter specification for larger temperature change. One should also note that the larger the expected temperature change is, the higher is the WTP. In contrast, the higher is τ, the upper bound on temperature change, the lower is the WTP. Society is more willing to pay when they are expecting worst outcomes to come and less willing, the less effective the environmental policy is. Finally, in all the specifications except from the additive, WTP increases as the initial growth rate decreases. The reason is that lowering g_0 lowers the entire trajectory for the consumption discount rate. This rate falls as ΔT increases but its starting value is $\delta + \eta g_0$. The damages from warming are initially small, making estimates of WTP highly dependent on the values for δ, η, and g_0 .

3.3 Uncertainty Limited to Temperature Change

The discussion above also applies to models with uncertainty. As an illustrative example, I will examine the case where there is uncertainty only with regards to temperature change: the results extent to economies with uncertainty over the economic impact too. Therefore, I assume that there is uncertainty over the trajectory of ΔT as represented by its cumulative distribution while the loss function is deterministic, with the parameters fixed at their mean values. I focus on the benchmark specification of Pindyck and the one with the additive loss function which allows for higher WTP. Table 2 shows $w^*(\tau)$ for several values of τ, setting $\eta = 2$, $\delta = 0$ and $g_0 = 0.015 - 0.025$. The first thing to observe is that in the benchmark specification, WTP is always below or just around 2% even for small values of τ. The values become even smaller as initial growth rate increases as the consumption discount rate becomes bigger. In contrast, in the additive specification WTP is in the range of 10-40 % and increases as initial growth increases. These are remarkably large values and one could wonder whether it is reasonable to accept that society would be willing to sacrifice around 30% of their lifetime income to prevent a temperature change of 2°C. However, it is indicative for the discussion of how different assumptions regarding the functional forms can have to radically different policy implications.

4 Policy Implications and Conclusions

The policy implications of these results are stark. As Pindyck states, in order to support a stringent abatement policy, one has to find values for the WTP around 2-3%. However, following his specification, the results show that WTP is below 2% even for small values of τ. In the case of no uncertainly, WTP to avoid any temperature change increases as the expected temperature change increases but it is still only around 6% when one expects $\Delta T = 10°C$ which is far beyond the temperature change needed to end human life as we know it. Although these estimates do not support the immediate adoption of a stringent GHG abatement policy, they do not imply that no abatement at all is optimal. For example, values of the WTP close to 2% is in the range of cost estimates for compliance with the Kyoto Protocol.

I therefore examine alternative specifications of the benchmark model. Employing a multiplicative damage function applied on the level instead of the growth of consumption leads to an increase in the WTP but only for large values of expected temperature change. It becomes higher though when we allow for an exponent of 3 or larger in the damage function as it better accounts for extreme events. However, the crucial assumption in these specifications is the substitutability between market and non-market consumption. If one assumes that the main impact of temperature change is on things that cannot be easily translated to material consumption such as health and biodiversity, then an additive rather than a multiplicative utility function has to be employed. My estimates show, that in this specification WTP gets remarkably high values even if expected temperature change is relatively small or the target temperature change is relatively big.

Although one could not strongly argue which is the right specification for the model, the main purpose of this paper is to show how seemingly small differences in modeling can have very different policy implications. Deep structural uncertainties inherent in the climate change science require a careful treatment in order for the analysis to lead to robust results. Moreover, a more realistic model that accounts for parametric as well as intrinsic uncertainty in the form of exogenous shocks and risks needs to be developed, if we are to safely answer the question of whether a stringent abatement policy needs to be employed.

5 Tables

	Pind. Model			*Exponential Bounding*			*Rational Bounding*			*Additive*		
ΔT	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025
2	0.011	0.007	0.005	0.006	0.003	0.002	0.005	0.003	0.002	0.086	0.11	0.135
4	0.023	0.014	0.009	0.02	0.013	0.009	0.018	0.013	0.009	0.272	0.332	0.384
6	0.035	0.022	0.014	0.044	0.029	0.021	0.04	0.028	0.021	0.457	0.528	0.583
8	0.048	0.029	0.019	0.077	0.053	0.038	0.069	0.049	0.037	0.51	0.665	0.713
10	0.06	0.036	0.024	0.119	0.083	0.06	0.103	0.075	0.056	0.700	0.757	0.795

Table 1: No Uncertainty, $\tau = 0$

	Pindyck			*Additive*		
τ	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025
0	0.019	0.011	0.007	0.311	0.376	0.429
2	0.011	0.007	0.005	0.280	0.338	0.385
4	0.006	0.004	0.002	0.194	0.234	0.268
6	0.003	0.002	0.001	0.108	0.13	0.148

Table 2: Uncertainty Limited to Temperature Change

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