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Tradable Permits in Cost–Benefit Analysis. A Numerical Illustration

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Abstract

There are different views with respect to the treatment of tradable permits for greenhouse gases in cost–benefit analysis. This note aims at illustrating numerically within a simple general equilibrium model how to treat tradable permits in economic evaluations of projects. The note looks at a cost–benefit rule for a large project providing a public good interpreted as a shortcut for infrastructure, using a fossil fuel and a renewable as inputs. The paper also evaluates a small or marginal project involving the same output and inputs. In addition, it illustrates the Samuelson condition for the optimal provision of the public good. The note is a supplement to CERE Working Paper No 2015:11 and SSE Working Paper in Economics No 2015:3. The model used here may also be useful in advanced courses to illustrate general equilibrium cost–benefit analysis.

Keywords: Cost–benefit analysis; greenhouse gases; tradable permits; general equilibrium; Samuelson condition; numerical illustration.

JEL-codes: H21; H23; H41; H43; I 30; L13.

1 Introduction

Tradable permits for greenhouse gases is an important tool in combating climate change. In order to emit one ton of gases a polluter must acquire a permit. There is a fixed number of permits, implying there is a cap on emissions. The question arises how to handle tradable permits in cost–benefit analysis. According to Johansson (2015), seven different major Manuals/Guides from North America, Europe, Australia, and Asia offer at least five different answers. In Johansson (2015) I discuss how to handle tradable permits in cost–benefit analysis, with special reference to the EU Emissions Trading System (EU ETS). This supplement aims at providing a simple numerical illustration of some basic results. The focus here is on the more complicated large-project case, while Johansson (2015) focuses on small or marginal projects; however, in Appendix B Johansson (2015) offers a brief discussion of the non-marginal case. How to assess a project using Hicksian expenditure functions is also discussed. In addition, this note briefly considers the optimal provision of a public good. The model is easily adapted so as to cover other evaluation scenarios than the one considered here. Therefore, the model could be useful in advanced courses to illustrate general equilibrium cost–benefit analysis.

2 A Simple General Equilibrium Model

There is just a single representative household, equipped with a simple utility function $U = \ln(x)+\ln(x_N)+\ln(z+1)-\ln(G)$, where x and x_N are commodities, z is a public good (in the counterfactual $z = 0$), and G denotes emissions of greenhouse gases. The indirect utility function, also the social welfare function in this representative household economy, is stated as:

$$V(p^x, y, z, G) = 2 \cdot \ln(y) - \ln(p^x) - 2 \cdot \ln(2) + \ln(z + 1) - \ln(G), \qquad (1)$$

where y is lump-sum income, and p^x is the price of x while $p_N = 1$. The household owns all firms and hence receive any profit income. There is also a (positive or negative) lump-sum tax. This tax is the difference between the cost of providing the public good (if $z > 0$) and the public sector's revenue from selling permits. Profits less the tax constitute y; refer to equation (5).

Assume there is an aggregate price-taking sector emitting greenhouse gases (in what follows termed the *Sector*) that are covered by a cap-and-trade system. It produces a commodity using a simple Cobb–Douglas technology:

$$x^s = 50 \cdot r^{1/3} e^{1/3}, \qquad (2)$$

where r is demand for a renewable input and e is demand for a fossil fuel covered by a permit system (perhaps indirectly through the electricity demanded by the sector and where the marginal plant uses a fossil fuel). As the sum of the exponents is 2/3, i.e., smaller than 1, the production function is strictly concave. Thus, the *Sector* has decreasing returns to scale so that its supply curve is upward-sloping. The *Sector*'s profit is:

$$\pi = p^x \cdot x^s - p^r \cdot r - (p^e + p) \cdot e, \tag{3}$$

where p^x is the output price, p^r is the price of the renewable, p^e is the energy price, and p is the permit price; as in Johansson (2015), the permit price is rescaled so that the cost of the permits can be added to the cost of the fossil fuel. In order to simplify the example, p^x and p^e are assumed to be determined in world markets and hence are considered to be exogenous, with $p^x = p^e = 1$. Moreover, the renewable is assumed to be produced using a constant returns to scale technology so that the supply is determined by aggregate demand. Demand functions for the renewable and permits are:

$$\begin{aligned} r &= \mathrm{k}\frac{(p^x)^3}{(p^r)^2 \cdot (p + p^e)} \\ e &= \mathrm{k}\frac{(p^x)^3}{(p + p^e)^2 \cdot p^r}, \end{aligned} \tag{4}$$

where k is a constant (equal to $125 \cdot 10^3/27$), e denotes emissions as well as energy, and for the sake of clarity all price symbols are maintained but in what follows $p^x = p^e = p^r = 1$. These constant price assumptions means no real loss of generality in the present context.

The public good is produced using the fossil fuel and the renewable resource as inputs. The public good is covered by the cap-and-trade system. The production technology is taken to be Leontief: $z = \min\{r^z, e^z\}$, where a superscript z refers to the public sector. Therefore, income is defined as follows:

$$y = \pi(.) + T = \pi(.) + p \cdot e^q - (p + p^e) \cdot e^z - 1 \cdot r^z, \tag{5}$$

where $T \gtreqless 0$ is a lump-sum payment/tax, and e^q is the given number of permits. Thus, any difference between the revenue from permits and the cost of producing the public good is handled in a lump-sum fashion. Note that the public sector earns revenue only by selling permits to the private sector, i.e., net revenue equals $p \cdot (e^q - e^z)$.

The model is general equilibrium, but in this note only the permit price is flexible (but implicitly the exchange rate adjusts so as to clear the current account although the considered projects are assumed to be so small relative to the size of the economy that this impact can be ignored; refer to equation (8)). This assumption is easily relaxed, but at the cost of increased computational complexity.

3 A Large Project

In this section, I consider such a large increase in the provision of the public good that it affects significantly the permit price. There are 50 permits available. Initially these are demanded by the *Sector* because $z^0 = 0$. Equilibrium in the permit market is reached when the permit price is such that $e(.) = 50$. This occurs when $p \approx 8.6225$. This is illustrated in Figure 1. Suppose now that the public sector decides to provide 10 units of the public good. This requires 10 permits because the production technology is Leontief. An equilibrium in the permit market occurs when the permit price is such that $e(.) + 10 = 50$, i.e., when $p \approx 9.75829$. At this price the *Sector* demands 40 permits. Thus, emissions within the "bubble" remains constant, i.e., the considered project has no impact on emissions. It is ignored here that the project may impact on emissions elsewhere (e.g., through the harvest of the fossil fuel).

[Figure 1 about here.]

Taking the partial derivative of the *Sector*'s profit function with respect to the permit price yields the negative of the demand function for the fossil fuel; alternatively, use equation (3) and the unconstrained Envelope Theorem, see Johansson and Kriström (2015). Hence, *Sector*'s loss of producer surplus can be obtained as the negative of the area to the left of the demand curve for the fossil fuel between initial and final prices of the composite commodity:

$$\Delta PS = -\int_{p^{c0}}^{p^{c1}} e(.)dp^c = -\int_{9.6225}^{10.75829} e(.)dp^c \approx -50.7937, \tag{6}$$

where $p^c = p^e + p$, and a superscript 0 (1) refers to the initial (final) equilibrium. In terms of Figure 2 it is the area to the left of the *Sector*'s demand curve between initial and final prices of the composite commodity, i.e., $p^{c0}\mathrm{AB}p^{c1}$. Alternatively, one could integrate with respect to the permit price and arrive at the same result. There are also other ways of estimating the loss of producer surplus. One way is to estimate the producer surplus as an area to the left of the supply curve for x between the choke price, where supply is equal to zero, and the output price $p^x = 1$ holding p at its final level, and deduct the area (producer surplus) obtained when p is held at its initial level. Still another variation is to simply calculate profits at the final permit price less profits at the initial permit price.

[Figure 2 about here.]

The public sector's revenue from permits decreases from $50 \cdot 8.6225$ to $40 \cdot 9.75829$, i.e., by 40.7934. In addition the public sector pays $1 \cdot 10$ for the fuel and $1 \cdot 10$ for the renewable input. Adding these three components to $-\Delta PS$ yields a total cost of producing the public good equal to about $\Delta C^T = 111.587$. Another way of arriving at this result is to estimate the cost as an area under an inverse demand function for the fossil fuel between initial and final demands:

$$\Delta C = -(\int_{e^0}^{e^1} \left(\frac{125 \cdot 10^3}{27 \cdot e}\right)^{1/2} de = \int_{50}^{40} \left(\frac{125 \cdot 10^3}{27 \cdot e}\right)^{1/2} de) \approx 101.587, \tag{7}$$

where the second line of equation (4) with $p^x = p^r = 1$ has been used to obtain the inverse demand curve for the composite commodity (i.e., $p^e + p$). ΔC in equation (7) corresponds to the shaded area in Figure 2, i.e., captures the loss of the marginal product of the composite commodity evaluated along the optimal path. This highlights what the cost of acquiring the fossil fuel reflects. Adding the cost of the renewable input, i.e., $1 \cdot 10$, to ΔC yields the total cost (111.587) of the considered increase in the provision of the public good. Using the initial (final) price of the composite commodity times the number of permits used by the public sector plus the cost of the renewable provides a quite good lower (upper) bound for ΔC^T or around 106.2 (117.6).

The reader may question that the derivation of ΔC^T is general equilibrium because one must account for induced price effects, in general. However, consider the partial derivative of the indirect social welfare function (1) with respect to p^x:

$$\frac{\partial V(.)}{\partial p^x} = V_y(.)[x^s(.) - x(.)], \tag{8}$$

where $V_y(.) = \partial V(.)/\partial y$ is the marginal utility of income. If the price clears the market, the derivative equals zero. Thus, in the more general case where there are many markets and flexible market prices, there will be many terms $0 \cdot dp^i = 0$, where $i = 1, 2, \ldots$, that don't show up in the final cost–benefit rule. If more than a single price (here p) adjust more than marginally one has to evaluate line integrals. This considerably complicates the analysis. Refer to Johansson and Kriström (2015, Ch. 6) for details.

In order to calculate the willingness-to-pay (WTP) for the considered project, one must use the social welfare function in equation (1) or a Hicksian expenditure function. The WTP is implicitly defined by the following equation:

$$2 \cdot \ln(y^1 - CV) + \ln(11) = 2 \cdot \ln(y^0), \tag{9}$$

where $y^0 = \pi^0 + p^0 \cdot 50 \approx 912.251$ and $y^1 = \pi^1 + p^1 \cdot 40 - p^e \cdot 10 - p^r \cdot 10 \approx 800.663$ (and all firms (including the producer of the renewable input) except the *Sector* make zero profits). It is easily verified that $CV \approx 525.609$. Emissions of greenhouse gases remain constant because there are a fixed number of permits: In both the factual and the counterfactual case emissions are equal to 50 units. Thus, CV represents the outcome of a cost–benefit analysis of an increase in the provision of the public good from $z = 0$ to $z = 10$ using a fossil fuel (under a cap-and-trade system) and a renewable as inputs.

The WTP for the public good is obtained as:

$$2 \cdot \ln(y^0 - CV^z) + \ln(11) = 2 \cdot \ln(y^0). \tag{10}$$

CV^z is equal to about 637.197. The difference between CV and CV^z is equal to the reduction in income ($\Delta y \approx -111.587$), i.e., to the total cost of providing 10 additional units of the public good. Thus the cost–benefit rule can be stated as follows:

$$\frac{\Delta V}{\overline{V}_y} = CV = CV^z + \Delta y = CV^z - \Delta C^T \tag{11}$$

where $\overline{V}_y$ is the marginal utility of income evaluated at an intermediate income $m \in [y^1 - CV, y^1]$; $\overline{V}_y = 2/m \approx 2/491.926$, and the marginal utility of income is continuous on the closed interval $[y^1 - CV, y^1]$. Refer to equation (B.2) in Johansson (2015) for further technical details.

4 Abatement Costs

Not much would be changed if there is an abatement technology. It simply adds a competitor to permits and hence causes a reduction of the permit price. However, emissions will remain unchanged, i.e., equal 50 units, as long as the cap is binding. To illustrate, if the *Sector* controls e^c units and its strictly convex control cost function is quadratic, i.e., $c(e^c) = (e^c)^2/2$, it is easily verified that the profit function can be stated as:

$$\begin{aligned}\pi(.) &= p^x \cdot x^s(.) - p^r \cdot r(.) - p^e \cdot e(.) - p \cdot (e(.) - p) - p^2/2 \\ &= p^x \cdot x^s(.) - p^r \cdot r(.) - (p^e + p) \cdot e(.) + p^2/2,\end{aligned} \tag{12}$$

where I have used the fact that the *Sector* will abate until the permit price equals the marginal abatement cost, i.e., $p = e^c$, and then turn to purchasing permits (implying that emissions as well as the number of permits demanded are given by $e^m = e(.) - e^c$). Similarly, the public sector may have access to an abatement technology and hence demand fewer than 10 permits, say, $X < 10$ permits. One would then arrive at a new equilibrium in the permit market where the private sector demands $50 - X$ permits and the public sector demands X permits, i.e., in total demand equals the fixed supply. If $X = 8$, the equilibrium permit price is 8.56828. The *Sector* will use 50.56828 units of the fossil fuel, control 8.56828 units of emissions, and hence purchase 42 permits. Thus, at the new equilibrium price demand for permits equals "supply". Note that $\partial\pi(.)/\partial p = -e^m(.) = -(e(.) - e^c) = -(e(.) - p)$, rather than being equal to $-e(.)$ as in equation (6), a fact that must be accounted for in calculating the loss of producer surplus.[1]

The only complication is that it seems difficult to come up with an inverted $e^m(.)$ function. However, one can use a variation of the Rule of Half, see de Rus (2010) or Johansson and Kriström (2015), to value the change:

$$\Delta PS + \Delta R \approx (42 - 50) \cdot (8.56828 + 7.939) \cdot \frac{1}{2} \approx -66.03, \tag{13}$$

where ΔR denotes the change in the government's revenue from permits. This approach produces a very close approximation, the error is around 0.08 monetary units. Add the cost of the fuel plus the cost of the renewable, i.e., 20, plus the project's abatement cost to arrive at a close approximation of the total project cost. If the $e^m(.)$ function is highly nonlinear one could decompose the

[1] In the initial situation $p^0 = 7.93897 = e^{c0}$, $e(.) = 57.93897$, and $e^{m0} = 50$. Moreover, $\Delta PS = -28.869$, and $\Delta C^T = 28.869 + 50 \cdot 7.93897 - 42 \cdot 8.56828 + 1 \cdot 10 + 1 \cdot 10 + e^{cz} = 85.95 + e^{cz}$, where e^{cz} is the public sector's abatement cost.

change in the number of permits into line segments and apply the Rule of Half to each trapezoid (British trapezium) and then sum the estimates. Alternatively, use pre-project and with-project permit prices to obtain bounds for the cost (63.5 and 68.5 plus 20 in both cases). Refer to Johansson (2015, Appendix A) for further technical details on how to model abatement costs.

5 A Marginal Project

Let us now turn to a marginal project. Suppose that it is evaluated at $z = 0$. Then the cost–benefit rule reads:

$$\begin{aligned}\frac{dV}{V_y} &= \frac{V_z}{V_y}dz - (p^e + p)de^z - p^r dr^z \\ &\approx \frac{1/1}{2/912.2511}dz - (1 + 8.6225)de^z - 1dr^z.\end{aligned} \tag{14}$$

where V_z/V_y is the marginal willingness-to-pay per additional unit of the public good. For technical details how to arrive at the cost–benefit rule in the first line, refer to Johansson (2015). Note that $p^e + p$ is equal to the value of the marginal product of the fossil fuel for the *Sector* evaluated at the initial optimum, i.e., $p^x \cdot 50 \cdot (1/3) \cdot r^{1/3} \cdot 50^{-2/3} = 9.6225$, where $r = 481.125$. This result neatly illustrates the interpretation of the cost of using a fossil fuel under an emissions trading system. If there is an abatement technology, one would have to add the marginal abatement cost to the cost of undertaking the considered project; abating emissions is costly but replaces permits that must be purchased in the market. If the marginal abatement cost is $7.93897 = p^0$, add the upper bound of the cost of controlling de^{zc} units, and change the number of permits from de^z to $de^z - de^{zc}$ (while the cost of acquiring the fuel continues to be $1 \cdot de^z$). If a control cost function for the considered activity is available, a more exact estimate of the abatement cost is obtained (assuming that the public sector abates such a quantity that the marginal control cost equals the permit price). It is straightforward to verify that the price of the composite commodity still is equal to the value of the marginal product of the fuel.

Next, suppose that the large project is evaluated as in (14), i.e., set $dz = de^z = dr^z = 10$ in equation (14). Then the CBA says that the societal surplus is around 4 455. This is a huge overestimation, due to using the WTP for the first unit of the public good to value all 10 units of the good. Suppose next that the project is evaluated at final levels:

$$\begin{aligned}\frac{dV}{V_y} &= \frac{V_z}{V_y}dz - (p^e + p)de^z - p^r dr^z \\ &\approx \frac{1/11}{2/800.663}dz - (1 + 9.7582)de^z - 1dr^z.\end{aligned} \tag{15}$$

Now, the social surplus is around 505.6. This is a slight underestimation of CV in equation (9). These illustrations show that using ruling prices/values may cause considerable overestimation of the social profitability of a large project.

6 Introducing an Expenditure Function

One could obtain the results using an expenditure function. It is easily verified that the Hicksian demand functions are:

$$x^H(.) = \frac{\exp^{\overline{V}/2} G^{1/2}}{(p^x \cdot (1+z))^{1/2}}$$
$$x_N^H(.) = \frac{\exp^{\overline{V}/2} G^{1/2} p^x}{(p^x \cdot (1+z))^{1/2}}, \tag{16}$$

where a superscript H refers to a Hicksian or income-compensated demand function, and $\overline{V}$ is the chosen reference level of utility. The expenditure function is defined as:

$$E(.) = p^x \cdot x^H(.) + x_N(.). \tag{17}$$

Taking the partial derivative of the expenditure function with respect to z one obtains:

$$\frac{\partial E(.)}{\partial z} = -\frac{(p^x)^{1/2} \cdot \exp^{\overline{V}/2} G^{1/2}}{(1+z)^{3/2}} \tag{18}$$

Evaluating the WTP for a marginal increase in the provision of the public good at $z = 0$ yields the same outcome as equation (14).[2] Integrating (18) between $z = 0$ and $z = 10$ yields the WTP for the considered large increase in the provision of the public good, i.e., CV^z in equation (10). Alternatively, simply use the expenditure function to calculate the change in expenditure. To obtain the net WTP (i.e., CV), deduct the absolute value of the change in income, i.e., $|\Delta y|$, which reflects the total cost of the project.

7 The Samuelson Condition

An additional issue is to determine the optimal provision of the public good. This is a quantity such that the marginal WTP for the good equals its marginal cost:

$$\frac{V_z}{V_y} = \frac{y}{2 \cdot (1+z)} = (1+p) + 1. \tag{19}$$

Recall that the assumed production technology is Leontief, i.e., the cost function is $g(p^r, p^e + p, z) = [(1+p)+1] \cdot z$. Hence, the right-hand side expression in (19) equals the marginal cost of providing z. The left-hand side expression as well as the middle expression expresses the representative individual's marginal WTP

[2] $p^x = 1$, and for any level of emissions of greenhouse gases G, but recalling that G affects the reference level of utility, which here is taken to be the one corresponding to $z = 0$: $\overline{V} = 2 \cdot \ln(912.2511) - 2 \cdot \ln(2) - \ln(G)$ so that $\exp^{\overline{V}/2} = 456.126/G^{1/2}$.

for the public good. In a society consisting of say, $H > 1$ identical individuals, multiply the marginal WTP by H.

The reader my want to relate this approach to the one in Drèze and Stern (1987), pedagogically presented in Florio (2014). Drawing on equation (2.17) in Drèze and Stern (1987), one can formulate a Lagrangian function:

$$L = H \cdot V(p^x, y^h, \overline{z}, G) - \lambda^e \cdot (e^q - e(.) - e^z) - \ldots - \lambda^z \cdot (\overline{z} - z), \tag{20}$$

where all markets but the permit market and the market for the public good are suppressed, there are $H \geq 1$ identical individuals, y^h is income of individual h ($h = 1, \ldots, H$), λ^e and λ^z are Lagrange multipliers, $\overline{z}$ is per capita consumption of the public good, and z is provision of the good. Taking the partial derivative of equation (20) with respect to $\overline{z}$ and setting the resulting expression equal to zero, one obtains:

$$H \cdot V_z = H \cdot \frac{1}{1+z} = \lambda^z. \tag{21}$$

In optimum, the aggregate marginal utility of the public good is equal to the shadow price (in utility units) of the good. Repeating the procedure for the provision of the public good, i.e., z, one obtains:

$$\lambda^z = V_y \cdot [(1+p) + 1] = V_y \cdot g_z, \tag{22}$$

where g_z is the marginal cost of providing the public good. Combining (21) and (22) provides the necessary condition for the optimal provision of the public good:

$$\frac{\lambda^z}{V_y} = (1+p) + 1 = g_z = H \cdot \frac{V_z}{V_y} = H \cdot \frac{y}{2 \cdot (1+z)}. \tag{23}$$

Thus, the good should be provided in such an amount that the shadow price of z, converted to monetary units by division by the marginal utility of income[3], equals the marginal cost of providing the public good, where permits are valued at the permit price. Equivalently, at the optimum, aggregate marginal willingness-to-pay for the public good equals the marginal cost of providing the good. Equation (19) provides the same answer (for $H = 1$). In both cases emissions within the "bubble" remains constant, while it is assumed that the project has no impact on emissions in the rest of the world. This rule resembles the one provided in Samuelson's (1954) classic paper.

Equations (19) and (23) reveal that there are three independent variables: z, y, and p. Hence, to obtain the optimal general equilibrium z-level, one must simultaneously solve for z, income y and the permit price p. Thus, three equations are needed (to obtain necessary conditions for an interior solution)

[3]The marginal utility of income equals the Lagrange multiplier or shadow price λ associated with the individual's budget constraint, evaluated at the optimum.

and here with $H = 1$:

$$\begin{aligned}
\frac{k}{(1+p)^2} &= 50 - z \\
y &= 50 \cdot (\frac{k}{(1+p)})^{1/3}(\frac{k}{(1+p)^2})^{1/3} - 1 \cdot \frac{k}{(1+p)} - (p+1)\cdot(50-z) \\
&\quad + p \cdot 50 - (1+p)\cdot z - 1 \cdot z \\
\frac{y}{2\cdot(1+z)} &= (1+p) + 1, \qquad (24)
\end{aligned}$$

where the fact that $e^z = e^r = z$ (due to the Leontief-assumption) is exploited. The first equation determines the permit price, conditional on z. The second equation yields income, conditional on p and z, and the third equation determines the optimal provision of the public good, conditional on p and y. The optimal provision of z is around 22.2024, with y around 645.273 and p around 11.9053. Figure 3 provides an illustration of how the provision of the public good affects welfare when the permit price and income are flexible but the number of permits is exogenous (and any impact on emissions in the rest of the world is ignored). The curve has been obtained by solving the first two lines of equation (24) for given z-levels and then using equation (1) to calculate social welfare.

[Figure 3 about here.]

8 Conclusions

The purpose of this note has been to illustrate how to handle a cap-and-trade system in evaluations of projects. The numerical illustration draws on a public good, but what type of project one is evaluating has no bearing on how to calculate the cost of using permits. The outcome of this exercise parallels the cost–benefit rules derived in Johansson (2015). If the project is small or marginal, value permits at their (forecast present value) market price. This price plus the fuel price reflects the value of the marginal product displaced elsewhere in the economy. If the project is large or non-marginal, the evaluation is slightly more complicated, but a quite obvious generalization of the cost–benefit rule obtained for a small or marginal project. Regardless of whether the project is small or large, it has no impact on total or aggregate emissions of greenhouse gases within the “bubble” (EU ETS, for example). Therefore, it would be a mistake to value permits at the global marginal damage cost; the damage cost of a zero increase in emissions is zero. Similarly, it would be a mistake to treat permits as a transfer within the private sector.

Acknowledgement

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Figures

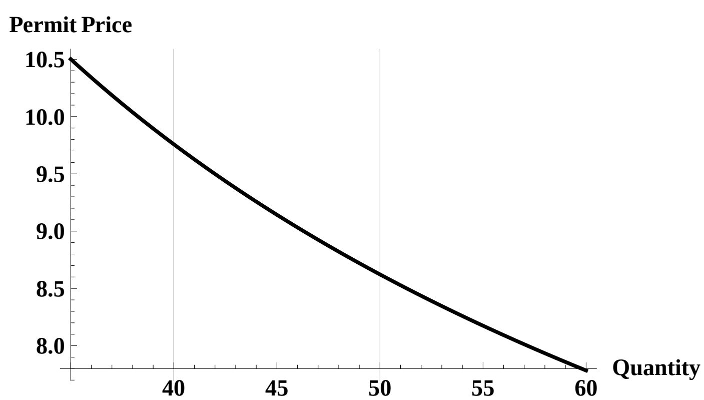

Figure 1: A segment of the *Sector*'s demand curve for permits. The vertical lines indicate at what prices the *Sector* demands 40 and 50 permits, respectively.

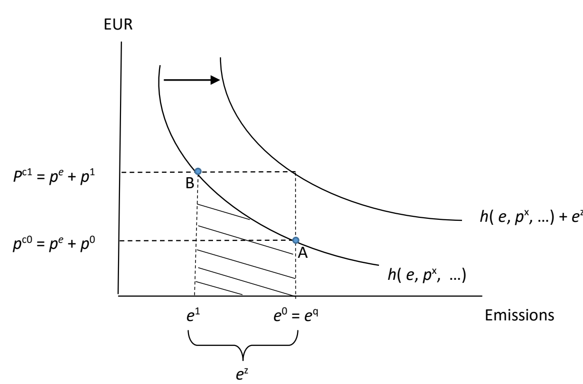

Figure 2: The area under the inverse demand curve, denoted $h(e, p^x \ldots)$, between e^0 (equal to the fixed number of permits e^q) and e^1 captures the public sector's cost of acquiring e^z units of a fossil fuel.

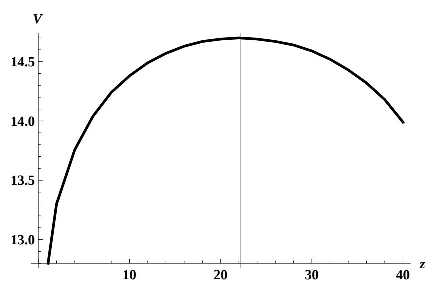

Figure 3: Social welfare V as a function of the provision of the public good z.