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Decisions under Risk Dispersion and Skewness

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Abstract

When people take decisions under risk, it is not only the expected utility that is important, but also the *shape* of the distribution of returns: clearly the *dispersion* is important, but also the *skewness*. For given mean and dispersion, decision-makers treat positively and negatively skewed prospects differently. This paper presents a new behaviourally-inspired model for decision making under risk, incorporating both dispersion and skewness. We run a horse-race of this new model against seven other models of decision-making under risk, and show that it outperforms many in terms of goodness of fit and, perhaps more importantly, predictive ability. It can incorporate the prominent anomalies of standard theory such as the Allais paradox, the valuation gap, and preference reversals.

JEL classification: D81.

Keywords: Decision under Risk; Anomalies; Valuation Gap; Preference Reversals; Allais Paradox; Skewness; Dispersion; Preference Functionals; Experiments; Pairwise Choice; Expected Utility; Non-Expected Utility; Stochastic Specifications.

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1. Introduction

We present a new story of decision-making under risk. Crucial to our story is that the decision-maker (henceforth DM) considers not only the expected utility of a lottery, but also the dispersion and skewness of the utilities. Our new theory explains how an individual values lotteries and hence takes decisions under risk. The theory is based on a behavioural description of the evaluation process.

In this theory this evaluation is thought of as a three-stage process: first, mirroring what Kahneman and Tversky write, the DM performs an editing process—in pairwise choice problems where one lottery first-order stochastically dominates the other, the DM chooses the dominating lottery; second, with respect to the remaining problems, the individual formulates an interval for the value of each lottery; finally he or she takes a weighted average of the extremes of this interval. Crucially, the interval depends upon the dispersion of the lottery, while the weights in the weighted average depend upon the skewness of the lottery and the optimism/pessimism of the individual. For expositional simplicity we initially restrict our analysis to pairwise choice problems.

Let us break this down into its three stages. The first, editing, stage is clear. As to the second stage, the literature suggests that many individuals find it difficult to state a precise Willingness-to-Pay (WTP) or Willingness-to-Accept (WTA) for a good (Bayrak and Kriström, 2016; Dubourg *et al*, 1994, 1997; Morrison, 1998). Studies show that if subjects are given the option of stating their subjective valuations in terms of a single amount or an interval, more than half of subjects prefer to state their valuations in terms of an interval (Banerjee and Shogren, 2014, Håkansson, 2008; Bayrak and Kriström, 2016). However, because of the problems in incentivising the true revelation of intervals (if they exist), this evidence does not *prove* that people think in terms of an interval, but only *suggests* it. But this seems a natural phenomenon: if asked to state their valuation for some lottery, individuals usually find it difficult to specify a precise number. Of course this depends upon the lottery: if it is a certainty, then there is no difficulty; if however, the lottery is risky then there is, and it becomes more difficult the more dispersed is the lottery. This is consistent with findings of Butler and Loomes (1988) and Cubitt *et al.*(2015), who conclude that, on the basis of their experimental evidence, the higher the variance of a lottery, the broader the imprecision range for a lottery.

Consider a lottery with pays either *x-d* or *x+d* each with probability one-half. The individual might, for example, say that the value is between *x-ad* and *x+ad* where *a<1*, and where *a* depends upon the confidence of the decision-maker. We formulate this more precisely shortly.

After the formation of an interval, the third stage sees the individual selecting a single value from the interval. This is done by taking a weighted average of the extremes of the interval, where the weights depend upon the *skewness* of the lottery and upon the optimism/pessimism of the decision-maker[1]. We shall explain in more detail in the next section.

Readers should note that we are not presenting a new normative theory, instead we focus on the descriptive side of the problem: we distill our new model from the accumulated experimental findings in the literature to explain observed behaviour.

The paper is structured as follows: section 2 formalises our theory; section 3 describes the 'horse race' that we conducted, comparing our model to seven others familiar in the literature, with our methodology and stochastic assumptions described in section 4. Section 5 details the results of the 'horse race'. Section 6 describes how our model explains some typical 'anomalies' found in the literature. Section 7 concludes. Other material can be found online.

[1] We note that there are studies interested in the relationship between skewness and preference. Kahneman and Tversky (1992) found that subjects exhibit risk-loving preferences for positively skewed lotteries and risk-averse preferences for negatively skewed lotteries. Others focus on long shots such as national lotteries and pursuing a career, which have a low probability of success but an extremely high prize for the case of success, for example, a career in the movie sector or in professional sports. Golec and Tamarkin (1998) find that people tend to favour the long-shot options in horse races with high prizes but low probabilities. Moreover, in national lotteries people are more concerned with the size of the prize rather than the expected value of the lottery, is interpreted as they are actually "buying a dream" which includes for example imagining how one can spend the prize and the joy of quitting one's job (Forrest *et al*, 2002; Garrett and Sobel, 1999).

2. Model

Let X be the set of outcomes (consequences) with elements denoted by x_i, $i=1...I$. The outcome set consists of real numbers designating amounts of money. The objects of choice are lotteries, which are probability distributions over the set X. A lottery is denoted by $z=\{x_1,p_1;...;x_I,p_I\}$, where $x_1<...<x_I$ and $p_1,...,p_I$ are the associated probabilities such that $p_i \geq 0$ and $\sum_{i=1}^{I} p_i = 1$. Since this paper focusses on decision under risk, the probabilities are taken as given by the DM. Let us denote by $\tilde{z}$ the *utilities* of the outcomes in the lottery and their associated probabilities: $\tilde{z}=\{u(x_1),p_1;...;u(x_I),p_I\}$. For notational convenience we denote the expected utility of the lottery by $E(\tilde{z})=\sum_{i=1}^{I} p_i u(x_i)$.

We consider first the situation when the DM is choosing between two lotteries; later we shall consider the changes necessary when the individual owns a lottery and is considering selling it, or when the individual does not own a lottery and is considering buying it.

The evaluation process described below applies only to the pairwise choices when neither lottery dominates the other. If one dominates the other, then the DM chooses the dominating lottery. This is taken care of in the first, the editing, phase of our model.

After this first stage, having taken decisions with respect to problems in which one lottery dominates the other, we now turn to problems where neither lottery dominates the other. For such lotteries the DM is thought of as formulating an interval for the value of each lottery, perceiving it as between *WEU(z)* and *BEU(z)*, which can be thought of as the Worst and the Best evaluations. This captures the idea that the DM is unable to attach a precise number to the value of a lottery, but instead comes up with an interval, saying that the lottery is worth somewhere between some lower number and some higher number. *WEU* and *BEU* are the utilities of these numbers. We posit that these are given by

$$WEU(z) = E(\tilde{z}) - kD(\tilde{z}) \quad (1)$$

$$BEU(z) = E(\tilde{z}) + kD(\tilde{z}) \quad (2)$$

These are centred on the expected utility of the lottery with the distance between them depending upon the dispersion $D(\tilde{z})$ of the utilities of the outcomes of the lottery, and upon a parameter *k* – which reflects the DM's uncertainty about his or her evaluations. $D(\tilde{z})$ denotes the mean absolute deviation of the utilities, defined by $D(\tilde{z}) = \sum_{i=1}^{I} p_i \,|\, u(x_i) - E(\tilde{z}) \,|$.

Our theory now posits that at the third stage the DM evaluates the lottery by taking a weighted average of the Worst and the Best. If we denote this valuation by *V(z),* it is given by:

$$V(z) = \alpha_{S(\tilde{z})} WEU(z) + [1 - \alpha_{S(\tilde{z})}] BEU(z) \quad (3)$$

Here $V(z)$ is a weighted average of the Worst and the Best expected utilities. $\alpha_{S(\tilde{z})} \in [0,1]$ is defined as the pessimism/optimism level of the individual, and this depends upon the skewness of the utilities in the lottery.

When we substitute (1) and (2) into (3) we get the following:

$$V(z) = E(\tilde{z}) + (1 - 2\alpha_{S(\tilde{z})}) kD(\tilde{z}) \quad (4)$$

Here all variables are measured in units of utility. Let us simplify this by putting $(1 - 2\alpha_{S(\tilde{z})}) = S(\tilde{z})$ thus getting the final form of our model:

$$V(z) = E(\tilde{z}) + kD(\tilde{z})S(\tilde{z}) \quad (5)$$

There are two key components[2] of the model, other than the editing phase. First the *dispersion* of (the utilities of) the lottery, $D(\tilde{z})$. Second the *skewness*[3] of (the utilities of) the lottery, $S(\tilde{z})$. Then there is the parameter *k*. Note that $D(\tilde{z})$ is necessarily positive while $S(\tilde{z})$ can be either positive or negative. The parameter *k* is individual-specific and could be either positive or negative. We discuss the implications below.

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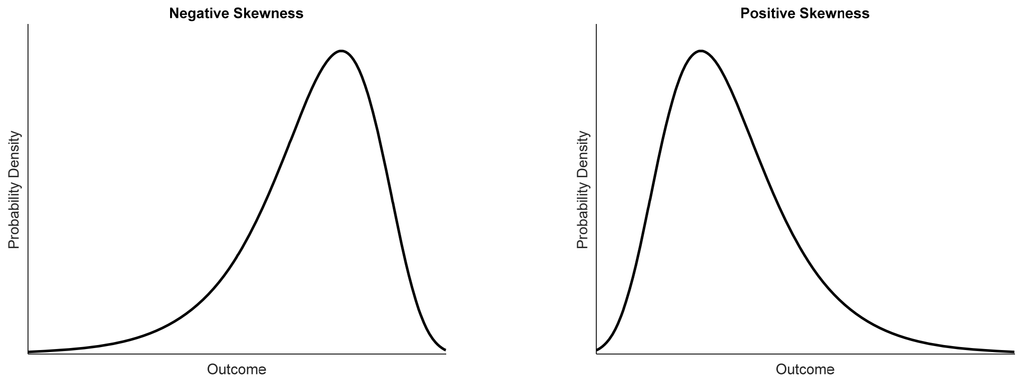
[2] We note that Hagen (1991) also proposes a model involving dispersion and skewness. Hagen incorporates these in an additive manner in the preference functional; they are seen as the source of extra utility and disutility, respectively. Instead our theory has behavioural motivations for the way that dispersion and skewness affect the preference functional.

[3] Which we measure using the distance measure: $\sum_{i=1}^{I}\sum_{j=1}^{I} p_i p_j \,|\, u_i - u_j \,| \Big/ \sum_{i=1}^{I}\sum_{j=1}^{I} p_i p_j \,|\, u_i + u_j \,|$ in keeping with our use of the mean absolute dispersion as our measure of dispersion.

Figure 1: Skewness Examples



If k is positive: As $D(\tilde{z})$ is positive, then how skewness affects the valuation depends upon the sign of the skewness. See the figure above. If the skewness is positive our theory implies that the lottery is valued more than it would be by its expected utility; moreover an increase in dispersion *in*creases the value of the lottery. This can represent the behaviour of an *optimistic* person whose attention is drawn to the possible high outcomes of the lottery. If the skewness is negative our theory implies that the lottery is valued less than it would be by its expected utility; moreover an increase in dispersion *de*creases the value of the lottery. This can represent the behaviour of a *pessimistic* person whose attention is drawn to the possible low outcomes of the lottery.

If k is negative: We get the reverse. As $D(\tilde{z})$ is positive, then how skewness affects the valuation depends upon the sign of the skewness. If the skewness is positive our theory implies that the lottery is valued less than it would be by its expected utility; moreover an increase in dispersion *de*creases the value of the lottery. This suggests a *pessimistic* person whose attention is drawn to the more likely low outcomes of the lottery. If the skewness is negative our theory implies that the lottery is valued more than it would be by its expected utility; moreover an increase in dispersion *in*creases the value of the lottery. This suggests an *optimistic* person whose attention is drawn to the more likely high outcomes of the lottery.

3. A horse race

We compare the goodness-of-fit and the predictive ability of this theory with some others standard in the literature. We use data from Hey (2001) which contains the pairwise choice responses of 53 individuals for the same 100 pairs of lotteries on five different days presented in different orders. The four monetary outcomes for the lotteries were -£25, £25, £75 and £125 respectively[4].

We consider the following preference functionals: Expected Utility theory (EU), Disappointment Aversion theory (DA), Prospective Reference theory (PR), Rank dependent expected utility theory with a Power weighting function (RP), Rank dependent expected utility theory with a Quiggin[5] weighting function (RQ), Salience Theory (ST) and Weighted Utility theory (WU). We test these against our Dispersion and Skewness theory (DS). Details of the preference functionals can be found in Hey (2001) and online, though we should comment briefly on our implementation of Salience Theory.

In the Hey (2001) experiment subjects were presented with two lotteries side by side and not juxtaposed as in Salience theory. So we have to make some assumption as to how subjects did the juxtapositioning. What we have done is the following. If the two lotteries are $X=\{x_1,p_1;x_2,p_2;x_3,p_3;x_4,p_4\}$ and $Y=\{y_1,q_1;y_2,q_2;y_3,q_3;y_4,q_4\}$, then we have assumed that the subjects consider the choice problem as over 16 'states of the world' leading to outcomes either x_i or y_j with probabilities p_iq_j (for $i=1,2,3,4$ and $j=1,2,3,4$). Now we can apply Salience Theory[6].

Our procedure is to estimate all eight models by maximum likelihood using GAUSS. We do this using the data in a variety of ways. We have 500 observations, collected in batches of 100 on 5 separate days. We do the following:

1. Estimate using the first 100 observations ("1st 100").
2. Estimate using the second 100 observations ("2nd 100").

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[4] There was a participation fee of £25.

[5] Strictly speaking, this is due to Kahneman and Tversky.

[6] Clearly this is not the only way that we can posit how subjects do the juxtapositioning.

3. Estimate using the third 100 observations (“3rd 100”).

4. Estimate using the fourth 100 observations (“4th 100”).

5. Estimate using the fifth 100 observations (“5th 100”).

6. Estimate using all 500 observations (“All 500”).

7. Estimate using the first 400 observations and predict on the last 100, using the estimates of the parameters from the first 400 (“1st 400”).

8. Estimate using the first 300 observations and predict on the last 200 using the estimates of the parameters from the first 300 (“1st 300”).

9. Estimate using the first 200 observations and predict on the last 300 using the estimates of the parameters from the first 200 (“1st 200”).

10. Estimate using the first 100 observations and predict on the last 400 using the estimates of the parameters from the first 100 (“1st 100”).

4. Methodology and stochastic assumptions

As noted above we fit the various models by maximum likelihood. To do this we need some assumptions about the stochastic nature of the data since it is abundantly clear that subjects make mistakes in experiments. We follow what Wilcox (2008) calls “a strong utility model”. The particular form of strong utility that we use first is what is sometimes called the Luce Model. In addition, since Wilcox (2008) reports that the stochastic specification may be more important than the preference functional, we also investigate the White Noise or Fechner story[7]. To explain these specifications, we need to give more detail. In the experiment there were four possible outcomes: -£25, £25, £75 and £125. All the models involve a utility function over the various outcomes. Such a function involves two normalisations. We normalise the utility of -£25 to be 0; the second normalisation comes through our strong utility story. In the Luce Model, in a pairwise choice between A and B where the value of A is V_A and the value of B is V_B, then the probability of A being chosen over B is given by $\frac{\exp(\lambda V_A)}{\exp(\lambda V_A)+\exp(\lambda V_B)}$ where λ is a

[7] Here we just report some of the results. All are online.

parameter representing the noisiness of the subject's responses; the larger is λ the noisier is the subject. We put $\lambda=1$; this is the second normalisation. The estimated values of the utilities of £25, £75 and £125 are therefore relative to this normalisation. The larger are the estimated values of the utilities of £25, £75 and £25, the noisier is the subject. The probability that A is chosen over B is thus $1/(1+\exp(V_B - V_A))$. In contrast, in the White Noise (Fechner) story this probability is given by 1-cdf(V_B-V_A) where cdf(.) is the cumulative distribution function of the unit normal.

This constitutes part of our stochastic specification. The other part is a tremble, which we apply only to our new preference functional as it is central to the theory. We apply it to the problems in the experiment where one lottery (first-order) stochastically dominates the other; our theory, in the editing phase, says that the DM chooses the dominating lottery. However, just as in the other problems, there is noise in decision-making and the DM might choose the dominated lottery. This we call a tremble, and we denote the probability of a tremble by *t*; there is no need to incorporate the Luce model for these problems. When we estimate our model, henceforth called DS, we apply the Luce Model (or the White Noise story) to all the non-dominating problems and a tremble to the dominating problems[8].

We report on two different versions, which differ depending upon the treatment of the DS tremble. First, since such trembles are few and far between (of the order of magnitude of just 3%), and hence difficult to estimate accurately even with up to 500 observations (of which there are just 6% dominating problems), we assume an exogenously given tremble probability. This we take to be 3%, following a suggestion from Nat Wilcox. This we call Version A. Then we have Version B where we estimate the tremble probability (using the obvious estimator as the percentage of violations of dominance in the dominating problems). This, of course, increases

[8] By 'dominating problems' we mean pairwise choice problems where one lottery (first-order) stochastically dominates the other; by 'non-dominating problems' we mean those where neither (first-order) stochastically dominates the other.

the number of parameters of the DS model, and hence penalises it in comparison with the other models.

5. Results

This section summarises our results. More detail can be found on the website. We measure performance both by goodness-of-fit and by predictive ability. We start by examining the results for Version A, in which case the DS tremble is exogenous.

We first count the percentage of times that each model comes first, either on the Akaike or on the Bayes information criterion, or in terms of predictive ability. The first two both penalise the goodness-of-fit – the maximised log-likelihood – by the number of parameters involved in the preference functional. EU has three parameters, WU has five and all the others have four. The results using the Akaike criterion are in the top part of Table 1a. We mark the 'winners' in **bold**. It will be seen that DS scores highly almost everywhere. If we use the Bayes criterion, we get the results in the middle of Table 1a: here DS does somewhat worse and EU scores more highly, mainly as a consequence of the Bayes criterion punishing more heavily degrees of freedom (the number of parameters).

We get a different picture on predictive ability. We measure this by fitting the models on a subset of the observations and using the estimated parameters to predict decisions on the remaining observations. We measure predictive ability by the log-likelihood of the prediction set using the parameters from the estimation set. Now DS is 'best' in all rows, as can be seen from the final four rows of Table 1a.

Table 1a: % of the time that each model comes first; Luce Model

	EU	DA	DS	PR	RP	RQ	ST	WU
Akaike Criterion								
1st 100	8	6	**23**	8	8	**23**	17	11
2nd 100	6	8	**30**	4	23	23	4	9
3rd 100	11	6	**25**	13	11	23	8	8
4th 100	8	2	**30**	13	15	17	2	13
5th 100	9	4	21	9	15	**25**	4	15
all 500	0	0	**55**	13	9	15	2	6
1st 400	0	0	**53**	13	11	15	2	6
1st 300	0	0	**45**	13	11	17	6	8
1st 200	0	4	**42**	9	13	19	4	9
Bayes Criterion								
1st 100	**28**	6	11	8	8	23	15	4
2nd 100	**34**	6	19	4	17	23	4	0
3rd 100	**26**	6	25	8	11	19	8	2
4th 100	25	2	**26**	11	13	19	2	2
5th 100	**28**	0	19	8	11	25	6	6
all 500	2	0	**55**	13	8	17	2	4
1st 400	2	0	**55**	13	9	15	4	2
1st 300	6	0	**42**	13	9	19	8	4
1st 200	19	4	**32**	9	8	19	8	2
Predictive Ability								
1st 400	2	6	**36**	9	11	15	9	11
1st 300	0	2	**47**	13	8	13	8	9
1st 200	0	6	**51**	11	13	6	6	8
1st 100	4	2	**45**	11	15	8	9	6

An alternative way of looking how ‘good’ models are is to look at the *average* ranking rather than at the number of times each model comes first. Why one might prefer to do this is that if one model comes first for half the subjects and last for the other half, while a second model is always second, one might prefer the latter. We start again with the Akaike criterion. Note here that ‘first’ is ranked 1 and ‘last’ is ranked 8, so that the lower the average ranking the better. Table 1b gives the detail. DS does well throughout.

Table 1b: Average Rankings; Luce Model

Akaike Criterion								
	EU	DA	DS	PR	RP	RQ	ST	WU
1st 100	4.7	5.4	**2.9**	4.3	5.1	3.4	5.3	4.9
2nd 100	4.6	5.1	**3.0**	4.5	4.5	3.3	6.0	4.7
3rd 100	4.6	5.4	**3.1**	4.3	4.8	3.2	5.5	4.9
4th 100	4.8	5.6	**3.2**	4.2	4.4	3.3	6.1	4.2
5th 100	4.5	5.5	**3.0**	4.2	4.5	3.4	6.1	4.5
all 500	5.8	5.6	**2.0**	4.6	4.9	3.2	6.0	3.8
1st 400	5.9	5.5	**2.2**	4.4	5.0	3.2	6.0	3.9
1st 300	5.6	5.4	**2.3**	4.6	5.1	3.2	5.8	4.1
1st 200	5.4	5.2	**2.5**	4.4	4.8	3.3	5.7	4.6
Bayes Criterion								
1st 100	3.3	5.4	**3.2**	4.3	5.1	3.4	5.3	6.0
2nd_100	**3.0**	5.0	3.3	4.6	4.5	3.4	5.9	6.2
3rd 100	**3.2**	5.3	**3.2**	4.5	4.8	**3.2**	5.6	6.0
4th 100	3.5	5.6	**3.3**	4.1	4.5	**3.3**	6.1	5.6
5th 100	**3.1**	5.5	**3.1**	4.2	4.6	3.5	6.0	5.8
all 500	4.7	5.6	**1.9**	4.5	5.1	3.1	6.0	5.1
1st 400	4.5	5.6	**2.1**	4.2	5.1	3.1	5.9	5.5
1st 300	4.4	5.5	**2.4**	4.5	5.2	3.1	5.7	5.5
1st 200	3.7	5.3	**2.7**	4.5	4.9	3.3	5.6	6.0
Predictive Ability								
1st 400	5.6	5.0	**2.4**	4.6	4.7	3.6	5.8	4.1
1st 300	5.7	4.9	**2.4**	4.3	5.0	3.4	6.0	4.1
1st 200	5.3	4.7	**2.2**	4.5	5.2	4.1	5.6	4.4
1st 100	5.0	4.5	**2.4**	4.4	4.7	4.2	5.5	5.2

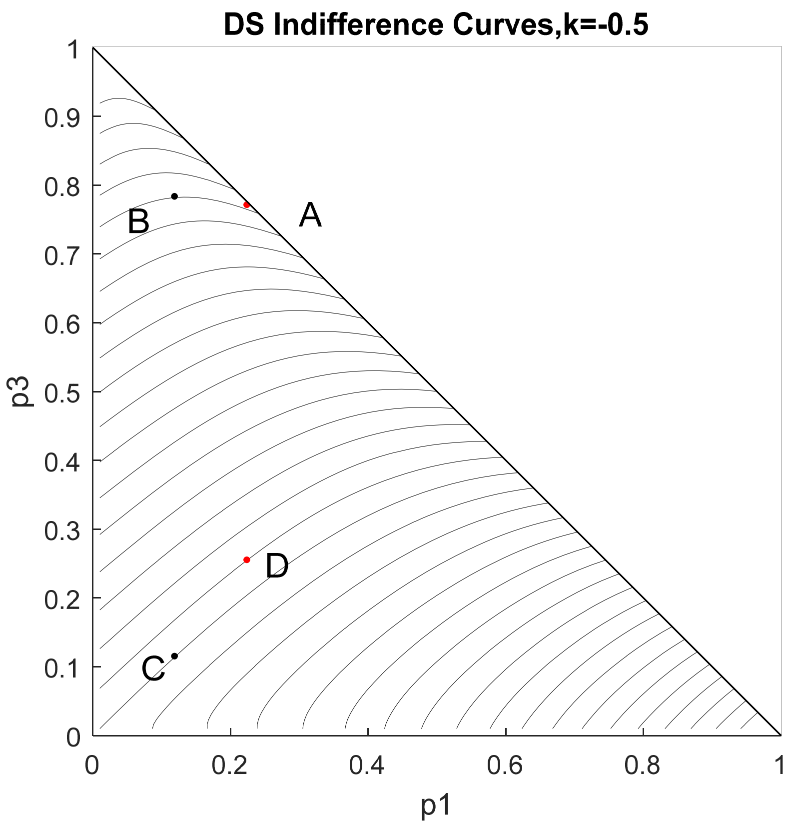
6. Anomalies

In this section we look at what can DS can tell us about the prominent 'anomalies' of standard economic theory[9]. Most of the 'anomalies' are situations in which behaviour is not consistent with that of EU. With EU, indifference curves in the Marschak-Machina Triangle (MMT) are parallel straight lines. This is not the case with non-expected utility theories, DS included. One apparent 'problem' with DS is that the indifference curves are not everywhere upward sloping when *k* is negative. In such cases stochastically dominating lotteries would appear to be the choice. But

[9] For a reader unfamiliar with the anomalies see online for a brief summary.

these are taken care of in the editing phase of DS. Examine the figure below, and consider the choice between A and B. It would appear that A would be chosen because it is on a higher indifference curve (utility increases from the bottom to top). But the editing phase says that B will be chosen because B dominates A. When neither dominates the other, for example, C and D, D, on the higher indifference curve, is chosen.

Figure 2: Dominance Pair (A,B) and Non-Dominance Pair (C,D)



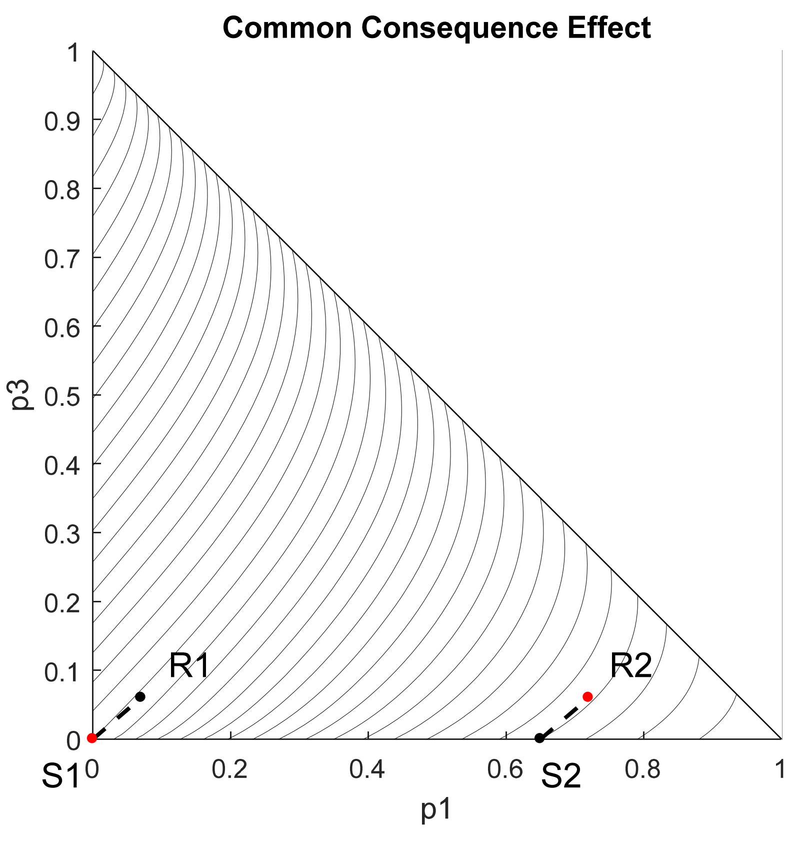
6.1. The common consequence and the common ratio effects

A common consequence problem is described by two choice problems between pair of lotteries, constructed in a specific way presented in Table 2. S2 and R2 are derived from S1 and S2 by moving probability *cc* (the 'common consequence') from x_2 to x_1. An individual whose preferences are compatible with EU would choose either 'S' or 'R' in **both** choice problems; common consequences added or subtracted to the two prospects should have no effect on the desirability of one prospect over the other, because the probabilities are incorporated in a linear way in EU[10]. Figure 3 superimposes the common consequence lotteries on a DS indifference map. Here S1 would be preferred to R1 and R2 to S2.

Table 2: Common Consequence Lotteries

Lottery	***p1***	***p2***	***p3***
S1	*0*	*1*	*0*
R1	*a*	*cc*	*1-a-cc*
S2	*cc*	*1-cc*	*0*
R2	*a+cc*	*0*	*1-a-cc*

Figure 3: Common Consequence Effect



A related phenomenon is the 'common ratio effect': There are two choice tasks and each task includes a pair of lotteries as shown in Table 3.

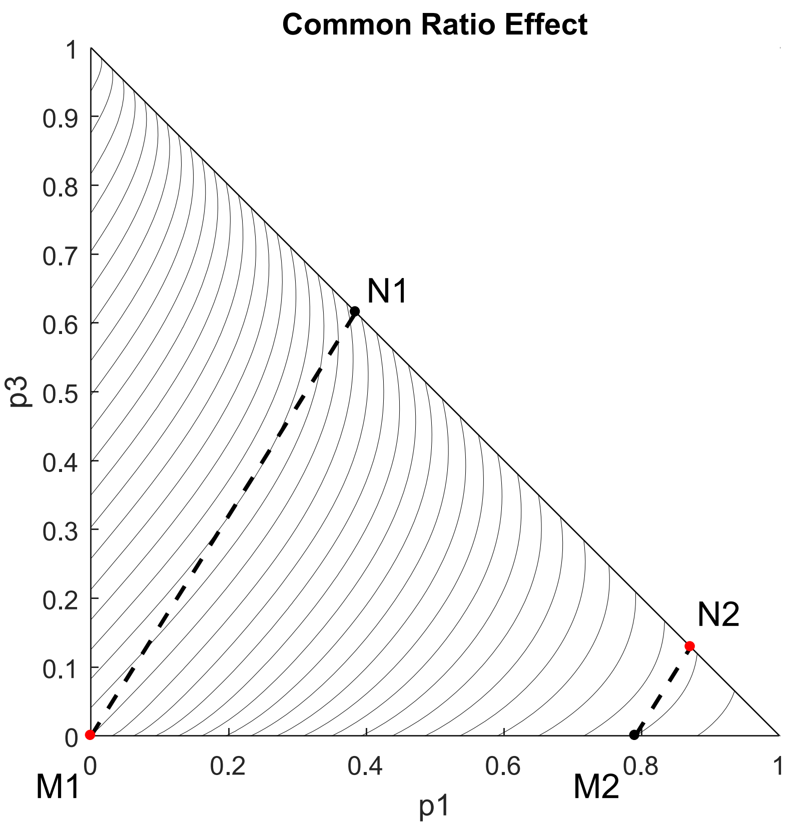
[10] Formally, under EU, putting $u(x_1)=0$, $u(x_2)=u$ and $u(x_3)=1$, S1 is preferred to R1 if and only if $u > cc.u +(1-b-cc)$, while S2 is preferred to R2 if and only if $u.(1-cc) > 1-b-cc$.

Table 3: Common Ratio Lotteries

Lottery	p_1	p_2	p_3
M1	*0*	*1*	*0*
N1	*1-a*	*0*	*a*
M2	*1-cr*	*cr*	*0*
N2	*1-a.cr*	*0*	*a.cr*

The common choice pattern of choosing M1 and N2 is inconsistent with the predictions of EU[11]. Figure 4 shows an example of such lotteries superimposed on DS indifference curves. Here M1 would be preferred to N1 and N2 to M2.

Figure 4: Common Ratio Effect



[11] Formally, under EU, putting $u(x_1)=0$, $u(x_2)=u$ and $u(x_3)=1$, M1 is preferred to N1 if and only if $u > a$, while M2 is preferred to N2 if and only if $u.cr > a.cr$.

6.2. The valuation gap

Our model implies that Willingness To Pay (WTP) may differ from Willingness To Accept (WTA). All the material above relates to a situation where the DM does not own a lottery and is considering buying one. If, however, the DM owns a lottery and is considering selling it, then things change. Now the Worst and Best situations change places. If the DM owns a lottery and sells it the worst thing that can happen is that the lottery turns out to have a *high* value, and therefore the DM would wish that he had sold it, and the best thing that can happen is that the lottery turns out to have a *low* value, and the DM is happy that he or she has sold it. So equation (3) above changes from

$$V(z) = \alpha_{S(\tilde{z})} WEU(z) + [1 - \alpha_{S(\tilde{z})}] BEU(z) \tag{3}$$

to

$$V(z) = [1 - \alpha_{S(\tilde{z})}] BEU(z) + \alpha_{S(\tilde{z})} WEU(z) \tag{6}$$

with the weights $\alpha_{S(\tilde{z})}$ and $[1 - \alpha_{S(\tilde{z})}]$ changing places. Inspection of (3) and (6) shows that valuations will change unless $\alpha_{S(\tilde{z})}$ = 0.5. So WTP may differ from WTA.

6.3. Preference Reversals

Unlike EU, DS allows for preference reversals under some conditions: Let P = *(x,p;0,1-p)* and \$ = *(y,q;0,1-q)* denote the P-bet and the \$-bet respectively, where *x* and *y* are the winning prizes with the associated probabilities *p* and *q* respectively. Preference reversals lotteries are conventionally constructed with the following properties: $x<y$, $p>q$ and the two bets are similar in their expected values. In a choice task individual prefers the P-bet over the \$-bet if the following holds, using equation 4:

$$E(\tilde{P}) + (1 - 2\cdot\alpha_{S(\tilde{P})})\cdot k \cdot D(\tilde{P}) > E(\tilde{\$}) + (1 - 2\cdot\alpha_{S(\tilde{\$})})\cdot k \cdot D(\tilde{\$}) \tag{7}$$

For simplicity, assume that the utility function is linear so $E(\tilde{P}) \approx E(\tilde{\$})$ since as mentioned above the bets are constructed in a way to have approximately equal expected values. We also know that the P-bet is negatively skewed giving a lower prize with a high probability and the \$-bet is positively skewed giving a higher prize with a low probability: $S(\tilde{\$}) > 0 > S(\tilde{P}) \Rightarrow \alpha_{S(\tilde{P})} > 0.5 > \alpha_{S(\tilde{\$})}$. Considering these inputs, the individual prefers the P-bet, that is, the left-hand side of (7) is higher than the right-hand side if *k* is negative.

On the other hand, in a selling task, as explained in Section 6.2, the weights attached to lowest and the highest expected utilities switches:

$$E(\tilde{P}) + (2 \cdot \alpha_{S(\tilde{P})} - 1) \cdot k \cdot D(\tilde{P}) < E(\tilde{\$}) + (2 \cdot \alpha_{S(\tilde{\$})} - 1) \cdot k \cdot D(\tilde{\$}) \quad (8)$$

The right-hand side will be higher if *k* is negative, implying that the individual states a higher WTA for the $-bet.

7. Conclusion

We present a new model of decision-making under risk which incorporates the dispersion and skewness of the returns from a lottery. We test this model against 7 others standard in the literature and show that it outperforms most in both explanatory and predictive ability. We also show that the new theory can explain some standard ‘anomalies’ such as the common consequence and common ratio effects and valuation gaps. The new model is parsimonious having only one parameter more[12] than EU. It is also behaviourally plausible, incorporating the fact that DMs take into account not only the expected utility of a lottery but also its dispersion and skewness: the whole *shape* of the distribution of returns is important to the DM.

[12] Depending upon the Version.

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Appendix A

Figure A1: DS indifference curves for different values of the parameter k

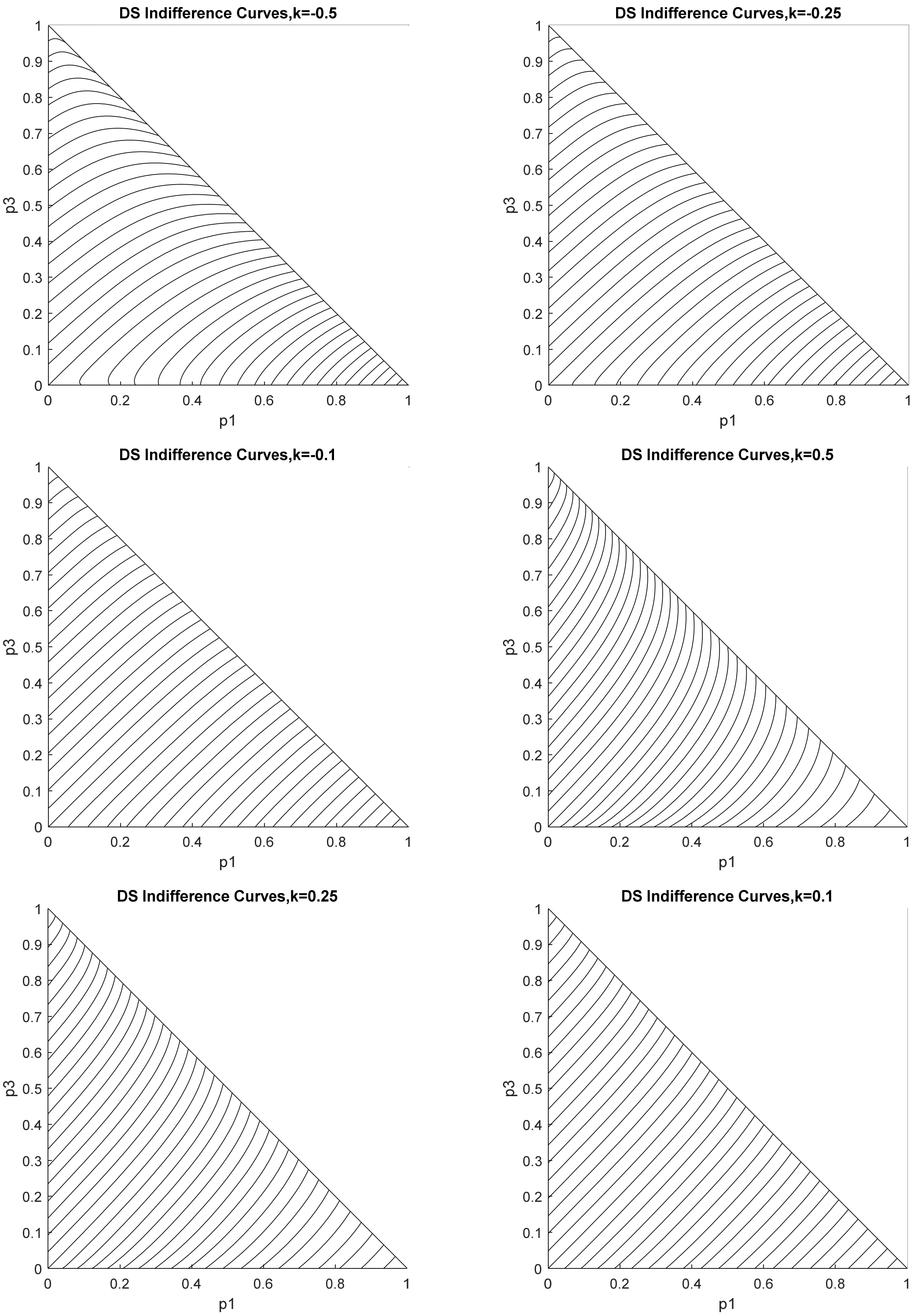
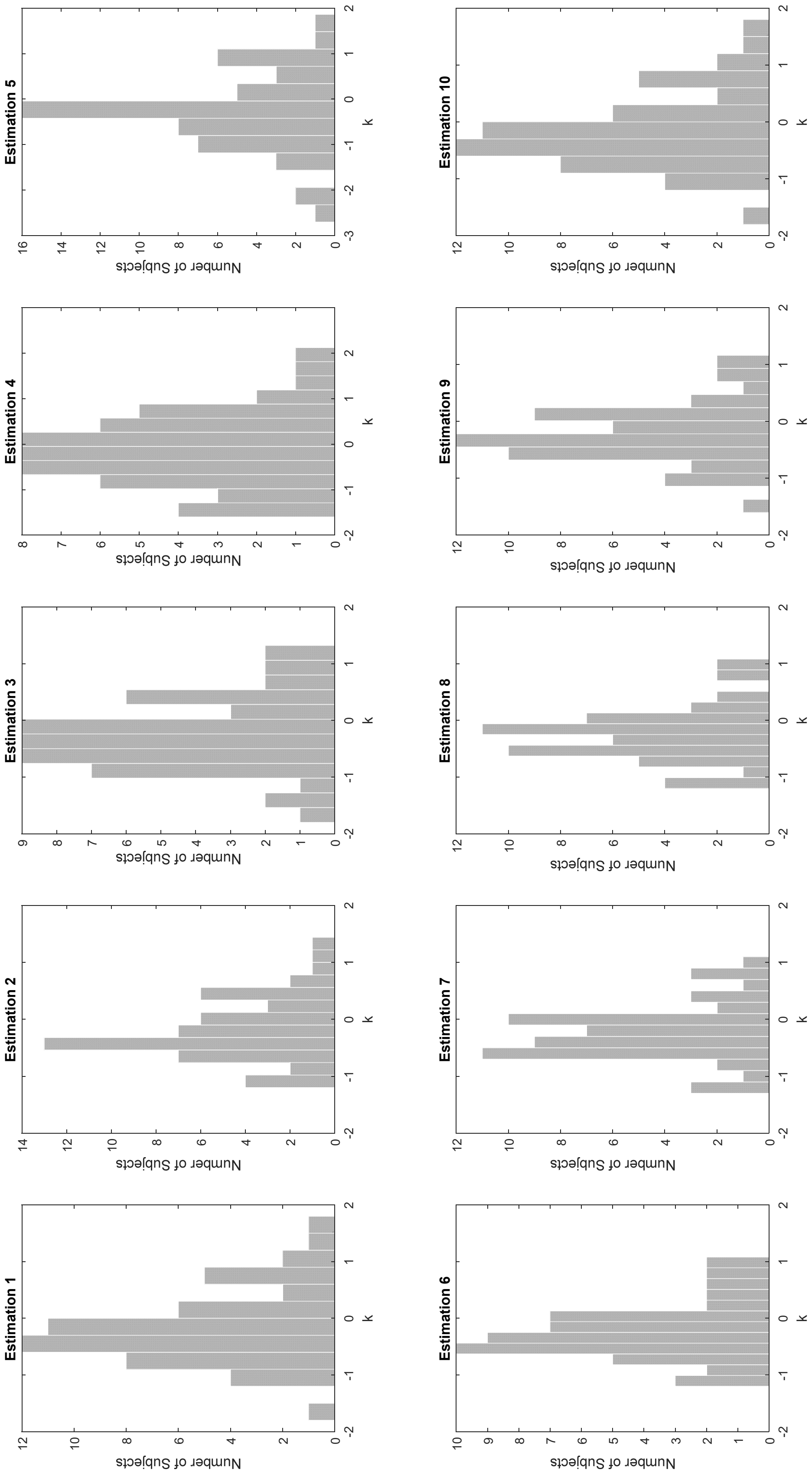


Figure A2: Histogram of k values (Exogenous Tremble with Luce Error)



Appendix B: Estimation Results of Version B with Luce Model

Table B1a: % of the times that each model comes first; Luce Model

Akaike Criterion	EU	DA	DS	PR	RP	RQ	ST	WU
1st 100	15	9	9	8	8	**23**	19	11
2nd 100	11	8	13	6	**26**	25	6	11
3rd 100	15	8	11	15	13	**25**	9	8
4th 100	13	4	13	13	15	**21**	4	17
5th 100	13	6	8	9	21	**26**	4	15
all 500	0	0	**53**	15	9	15	2	6
1st 400	0	2	**47**	13	11	17	2	8
1st 300	0	4	**38**	13	13	19	6	8
1st 200	2	6	**34**	9	13	21	4	11
Bayes Criterion								
1st 100	**28**	8	8	8	8	23	15	6
2nd 100	**45**	6	4	4	17	26	4	0
3rd 100	**28**	9	6	9	13	21	9	8
4th 100	**30**	6	6	11	13	23	4	8
5th 100	**32**	4	4	8	15	26	6	8
all 500	11	0	**38**	17	9	17	4	4
1st 400	11	2	**30**	17	9	19	8	4
1st 300	17	6	15	13	11	**26**	8	4
1st 200	**30**	6	13	9	8	23	9	2
Predictive Ability								
1st 400	2	6	**36**	9	11	17	9	9
1st 300	0	2	**47**	13	8	13	8	9
1st 200	0	6	**51**	11	13	6	6	8
1st 100	4	2	**45**	11	15	8	9	6

Table B1b: Average Rankings; Luce Model

Akaike Criterion								
	EU	DA	DS	PR	RP	RQ	ST	WU
1st 100	4.4	5.1	4.3	4.1	4.9	**3.1**	5.3	4.7
2nd_100	4.3	4.9	4.5	4.3	4.3	**3.1**	5.8	4.4
3rd 100	4.4	5.1	4.4	4.1	4.6	**3.0**	5.4	4.8
4th 100	4.6	5.4	4.2	4.0	4.3	**3.2**	6.0	4.1
5th 100	4.2	5.3	4.2	4.0	4.4	**3.3**	6.1	4.3
all 500	5.8	5.6	**2.1**	4.5	4.9	3.2	6.0	3.8
1st 400	5.9	5.5	**2.3**	4.3	5.0	3.1	6.0	3.9
1st 300	5.5	5.3	**2.6**	4.5	5.0	3.1	5.8	4.1
1st 200	5.3	5.2	**3.0**	4.4	4.8	3.2	5.7	4.6
Bayes Criterion								
1st 100	3.2	4.9	5.7	3.8	4.6	**3.0**	5.2	5.7
2nd 100	**2.7**	4.5	6.0	4.1	4.2	2.9	5.6	5.8
3rd 100	2.9	4.7	5.7	4.1	4.3	**2.8**	5.4	5.7
4th 100	3.2	5.1	5.5	3.7	4.2	**2.9**	5.9	5.3
5th 100	**2.8**	5.1	5.7	3.7	4.2	3.1	5.8	5.3
all 500	4.5	5.6	**2.6**	4.3	5.1	3.0	5.9	5.0
1st 400	4.2	5.5	3.1	4.1	5.0	**2.9**	5.9	5.3
1st 300	4.0	5.2	3.8	4.3	5.0	**2.8**	5.6	5.4
1st 200	3.4	4.9	4.8	4.1	4.7	**2.9**	5.4	5.7
Predictive Ability								
1st 400	5.6	5.0	**2.4**	4.6	4.7	3.6	5.9	4.1
1st 300	5.7	4.9	**2.3**	4.3	5.0	3.5	6.0	4.1
1st 200	5.3	4.7	**2.2**	4.5	5.2	4.1	5.6	4.4
1st 100	5.0	4.5	**2.3**	4.4	4.7	4.2	5.5	5.2