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Dynamic CGE-model with heterogeneous forest biomass: Applications to climate policy.☆

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Abstract

This study introduces a framework for modeling a renewable forest biomass stock interacting with economic sectors in a competitive economy. The equilibrium is formulated as a mixed complementary problem (which explicitly represents weak inequalities and complementarity among decision variables and equilibrium conditions). The complementarity format permits detailed modeling of the growth and harvest of a biomass stock together with a second-best characterization of the overall economy. First the complementarity features of economic equilibrium and its integration with an ecological representation of the biomass stock are provided. Then a stylized numerical example of a dynamic computable general equilibrium model is presented. Finally, illustrative applications of the model for gauging the likely effects of environmental subsidies and taxes intended to promote increases CO_2 storage in forest biomass are given, the results are discussed.

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1. Introduction

This study presents a dynamic computable general equilibrium (CGE) model that combines (in richer detail than similar, previous models) a general equilibrium description of an economy and an ecological model describing changes in forest biomass. Its main intended uses include modeling the likely effects of economic policy measures that could be introduced to foster carbon sequestration. For such purposes, a key parameter is the total biomass present in forest stocks, which can be fairly easily estimated. However, in order to model changes in total biomass, knowledge of other ecological aspects (notably the age-class distribution of the trees and rotation lengths) must be known (Getz and Haight, 1989). Therefore, the ecological model of the forest biomass stock presented here has a detailed age-structured representation of growth and harvests, interlinked with the economy. Harvests and demand for forest products are determined endogenously in an inter-temporally consistent way.

The CGE is formulated as a mixed complementarity problem (Mathiesen, 1985). The complementarity format permits detailed modeling of the growth and harvest of a biomass stock with a second-best characterization of the overall economy.

Beyond the direct integration of ecology and economy, the complementarity representation readily accommodates income effects and important second-best characteristics such as tax distortions or market failures (externalities). The utilization of weak inequalities allows modeling of problems where, for at least some of these inequalities, we do not know a priori which will hold as strict inequalities and which will hold with equality at an equilibrium. For example, inequalities may appear if production of a commodity is specified by alternative technical processes, or the economic problem under consideration may involve various types of institutional constraints on prices or quantities. These features of the complementarity format makes it particularly valuable in policy analysis.

The policy instruments included are taxes, subsidies and tariffs. Questions regarding the value of carbon sequestration and the cost of setting aside specific parts of forest land for recreation can also be addressed. Further, it is possible to impose restrictions on forest harvests, in accordance with prevailing regulations. Potential applications of the model are illustrated here by simulations of scenarios incorporating policies intended to increase CO_2 sequestration in the forest biomass, and thus contribute to efforts to mitigate anticipated global climate change (Van Kooten et al., 1995; Gong and Kriström, 1999).

The paper proceeds as follow. Section 2 offers a review of relevant published studies. Section 3 provides a formal exposition of the complementarity features of economic static equilibrium, and section 4 a description of the ecological forest growth part of the model. Section 5 introduces the stylized model, integrating dynamic forest biomass growth and harvest activity into economic modeling. Simulations are presented and discussed in section 6, and conclusions in section 7.

2. Literature review

The use of Computable General Equilibrium (CGE) models in analysis of the forest sector has been motivated by the importance of links between the forest sector and the rest of the economy (Haynes et al., 1995). In regions where the forest sector is an important contributor to employment and gross domestic product, the effect of changes in the forest sector on the economy may be of significant interest. For example, Binkley et al. (1994) used a CGE model to analyze the economic impact of reductions in the annual allowable cut in the Canadian province of British Columbia, where the forest industry is a major component of the economy. In addition, the Global Trade Assessment Project (GTAP) model has been used in an Asia-Pacific Economic Cooperation (APEC) study to assess the effects of removing specific non-tariff barriers to trade in forest products on a country's gross domestic product, welfare, and trade (New Zealand Forest Research Institute, 1999).

In other cases, partial equilibrium (PE) analysis has been applied. The CINTRAFOR Global Trade Model (CGTM) describes forest growth, wood supply, processing capacity and final demand. Market equilibrium is solved on a period-by-period basis with inter-period changes in forest inventory as a dynamic element. The CGTM has been applied to numerous forest sector issues, e.g., Perez-Garcia (1994), Perez-Garcia (1995), and Eastin et al. (2002). Detailed descriptions of the CGTM are presented in Kallio et al. (1987) and Cardellichio (1989). Another example of a PE model is the Global Forest Product Model (GFPM) (Buongiorno, 2003), which integrates timber supply, processing industry parameters, product demand and trade. For each year equilibrium is computed, while year-by-year changes are simulated by recursive programming. Both CGTM and GFPM are designed as policy analysis tools, but they do not attempt to predict the feedback effects of changes in either the forest sector or forest stands on the rest of the economy. Nor do they attempt to optimize the forest sector over the planning horizon. Yet another example of a partial equilibrium model is the Timber Assessment Market Model (TAMM) (Adams and Haynes, 1980), which focuses mostly on North America, but has been used to analyze international issues (Adams and Haynes, 1996). The two main components of the TAMM are a market model and an inventory projection module. The market model covers supply and demand for wood over regions and sectors. Pulp fiber requirements and projections of forest inventory and forest growth are exogenous inputs to the model. A drawback of TAMM is that the spatial equilibrium is found by "reactive programming", which makes it difficult to represent policy scenarios involving constraints on endogenous variables (Adams and Haynes, 1996).

The CGE and PE models discussed in the previous paragraph incorporate similar detail regarding the supply and demand sides of the forestry sector, lacking age-structured description of the biomass stock. Another feature of these models are that they are static, or have dynamic elements that link each period's solution, such as CGTM, GFPM and TAMM, but do not satisfy optimality in an inter-temporal sense.

The Timber Supply Model (TSM) (Sedjo and Lyon, 1998), on the other hand, was developed to study the transition of the world's forest from old-growth to plantation-grown industrial forests, and focuses on global timber supply. The modeling approach uses control theory to determine the inter-temporal optimal transition. The TSM is a dynamic model that is intended to describe the wood supply sector accurately, but it cannot be integrated with a second-best representation of the overall economy since it excludes demand side interactions.

3. Complementarity in a competitive market equilibrium

This section provides an introduction to the mixed complementarity problem (MCP), and relates the MCP format to the competitive economic equilibrium. The purpose is primarily pedagogic, but the difference between this section's static equilibrium and the inherently dynamic equilibrium (due to the nature of biomass growth) presented later in the paper, is not overwhelming. As will be shown, the intra-period equations can be applied to the scheme described in this section, and the few additional inter-period equations required can be readily incorporated.

Mathematically, the MCP is a square system of functional relations, $\mathbf{f}(\mathbf{z})$, and variables, $\mathbf{z}$, which can be stated as:

Find $\mathbf{z}$ such that:

$f_i(\mathbf{z}) = 0$ and $lo_i \leq z_i \leq up_i$

or

$f_i(\mathbf{z}) \geq 0$ and $z_i = ll_i$

or

$f_i(\mathbf{z}) \leq 0$ and $z_i = up_i$

where:

$\mathbf{z}$	is a vector of variables to be found
$\mathbf{f}(\mathbf{z})$	is a (possibly nonlinear) vector-valued function
up and **lo**	are vectors of upper and lower bounds on the variables, where elements in **lo** may be $-\infty$ and elements in **up** may be $+\infty$.

When modeling an economic equilibrium the variables typically describe activity levels and prices that are non-negative, see Mathiesen (1985), hence lower bounds are at zero ($\mathbf{lo} = \mathbf{0}$), upper bounds at infinity ($\mathbf{up} = \infty$), and the system of relations reduces to:

Find $\mathbf{z}$ such that:

$f_i(\mathbf{z}) = 0$ and $z_i \geq 0$

or

$f_i(\mathbf{z}) \geq 0$ and $z_i = 0$

The dimensions of $\mathbf{z}$ and $\mathbf{f}(\mathbf{z})$ are equal, producing a square system.

The problem can be written compactly as:

$$\mathbf{f}(\mathbf{z}) \geq \mathbf{0} \perp \mathbf{z} \geq \mathbf{0} \tag{3.1}$$

where the $\perp$ ("perpendicular to") symbol indicates pair-wise complementarity between the vector-valued elements of function $\mathbf{f}(\mathbf{z})$ and the variables $\mathbf{z}$. Note that the MCP does not have an objective function as in an optimization problem.

Consider a closed competitive market economy, Arrow and Debreu (1954), were individuals are price takers and engage in cost-minimization behavior. There are n tradable commodities (goods and factors) in the economy, some of which at least are produced in m (positive homogeneous) non-increasing return to scale (NIRS) production activities. There are h households, each with an initial endowment, w, of commodities. The decision variables of the economy can be classified into four categories:

- $\mathbf{p}$ is a vector ((n x 1) with index i) in prices for all goods and factors,
- $\mathbf{y}$ denotes a vector ((m x 1) with index j) of activity levels of NIRS production activities,
- $\mathbf{m}$ signifies a vector ((h x 1) with index k) of income levels for households,
- $\mathbf{u}$ represents a vector ((h x 1) with index k) of utility index for households, and
- $\mathbf{p}^u$ indicates a vector ((h x 1) with index k) of prices of the utility indexes.

The original formulation by Mathiesen (1985) contained only prices of goods and factors, $\mathbf{p}$, and activity levels, $\mathbf{y}$. The variables $\mathbf{m}$, $\mathbf{u}$, and $\mathbf{p}^u$ are introduced for convenience, but could be taken away.

The competitive market equilibrium is characterized by the following five conditions:

- Zero profit: implies that no economic activity earns positive economic profit:

$$-\pi_j(\mathbf{p}) = c_j(\mathbf{p}) - r_j(\mathbf{p}) \geq 0, \ \forall j \tag{3.2}$$

where:

$$c_j(\mathbf{p}) \equiv \min_{\mathbf{a}_j} \left\{ \sum_i p_i a_{i,j} \left| f_j\left(\cdot\right) = 1 \right. \right\} \quad \text{and}$$
$$r_j(\mathbf{p}) \equiv \max_{\mathbf{b}_j} \left\{ \sum_i p_i b_{i,j} \left| g_j\left(\cdot\right) = 1 \right. \right\}$$

defines the cost and revenue functions per unit activity level of activity j, resulting from cost-minimization behavior,

$\mathbf{a}_j$	denotes the feasible commodity input bundle per unit activity level of activity j,
$\mathbf{b}_j$	denotes the feasible commodity output bundle per unit activity level of activity j,
$f_j, \ g_j$	describes the feasible input- and output-combinations of production in activity j,
$\pi_i(\mathbf{p})$	denotes the unit-profit function for NIRS production activities,

- Market clearance: requires that no commodity is in excess demand, or equivalently, excess supply is non-negative[1]:

$$\xi_i(\mathbf{p}, \mathbf{y}) = \sum_j y_j \frac{\partial \pi_j(\mathbf{p})}{\partial p_i} + \sum_k \left[w_{i,k} - \frac{\partial e_k(\mathbf{p})}{\partial p_i} m_k \right] \geq 0, \; \forall i \qquad (3.3)$$

where:

$$e_k(\mathbf{p}) \equiv \underset{\mathbf{x}_k}{min} \left\{ \sum_i p_i x_{i,k} \, | u_k(\mathbf{x}_k) = 1 \right\}$$

[1]Note that from the assumption of linear homogeneity of utility we have $e_k(\mathbf{p}, m_k) = e_k(\mathbf{p}) m_k$.

	defines the compensated expenditure per unit utility for good i by household k resulting from expenditure-minimization behavior,
$\mathbf{x}_k$	denotes the consumption commodity bundle per unit utility for household k,
$u_k(\cdot)$	is a linear homogeneous utility function resulting from any homothetic preference ordering system,
m_k	indicates the income level of household k,
$w_{i,k}$	indicates the initial non-negative endowment of commodity i by household k,
$\frac{\partial e_k(\mathbf{p})}{\partial p_i}$	indicates the compensated demand for good i per unit utility of household k,
$\frac{\partial \pi_j(\mathbf{p})}{\partial p_i}$	is the supply of good i per unit operation of activity j,
$\xi_i(\mathbf{p}, \mathbf{y})$	represents the excess supply function for commodity i.

- Income balance: This is not a condition for equilibrium but rather a definition, introduced for convenience, of the income, m_k, for each household as the sum of the value of initial endowments:

$$m_k \equiv \sum_i p_i w_{i,k}, \ \forall k \tag{3.4}$$

- Zero profit (utility): This constraint, together with constraint (3.6), is introduced to control the levels of the variables $\mathbf{u}$ and $\mathbf{p}^u$. The constraint is not necessary for the equilibrium but introduced for convenience and states that unit expenditure equals the marginal price of utility:

$$e_k(\mathbf{p}) = p_k^u, \ \forall k \tag{3.5}$$

- Market clearance (utility): This constraint, together with constraint (3.5), is introduced to control the levels of the variables $\mathbf{u}$ and $\mathbf{p}^u$. The constraint is not necessary for the equilibrium but introduced for convenience[2]:

$$u_k = \frac{m_k}{p_k^u}, \ \forall k \tag{3.6}$$

- Irreversibility: All activities are operated at non-negative levels:

$$\begin{aligned} \mathbf{y} &\geq \mathbf{0} \\ \mathbf{u} &\geq \mathbf{0} \end{aligned} \tag{3.7}$$

- Free disposal: Prices remain non-negative:

$$\begin{aligned} \mathbf{p} \geq \mathbf{0} \\ \mathbf{p}^u \geq \mathbf{0} \end{aligned} \tag{3.8}$$

- Non-satiation: Assuming that underlying utility functions exhibit non-

[2]Note that with linear homogeneous utility, the indirect utility function can be written $V(\mathbf{p}, m) = \frac{m}{e(\mathbf{p})}$.

satiation, household expenditure will exhaust income, hence:

$$\sum_i p_i \left(w_{i,k} - \frac{\partial e_k(\mathbf{p})}{\partial p_i} m_k \right) = 0, \ \forall k \qquad (3.9)$$

Walras' law together with conditions (3.3) and (3.8) imply:

$$p_i \xi_i(\mathbf{p}, \mathbf{y}) = 0, \forall i \qquad (3.10)$$

This means that, in equilibrium, a commodity in excess supply must have zero price, and for a commodity with positive price the market must clear.

From conditions (3.3) and (3.9) and equation (3.10) we obtain[3]:

$$\sum_i p_i \xi_i(\mathbf{p}, \mathbf{y}) = 0 \qquad (3.11a)$$

$$\sum_i p_i \left[\sum_j y_j \frac{\partial \pi_j(\mathbf{p})}{\partial p_i} + \sum_k \left[w_{i,k} - \frac{\partial e_k(\mathbf{p})}{\partial p_i} m_k \right] \right] = 0 \qquad (3.11b)$$

$$\sum_j y_j \sum_i p_i \frac{\partial \pi_j(\mathbf{p})}{\partial p_i} + \sum_k \left[\sum_i p_i \left[w_{i,k} - \frac{\partial e_k(\mathbf{p})}{\partial p_i} m_k \right] \right] = 0 \qquad (3.11c)$$

$$\sum_j \lambda_j y_j \pi_j(\mathbf{p}) = 0 \qquad (3.11d)$$

$$\sum_j y_j \pi_j(\mathbf{p}) = 0 \qquad (3.11e)$$

[3]With the assumption of NIRS, unit profit functions are positive homogeneous in prices and by Euler's homogeneous function theorem $\lambda_j \pi_j(p) = \sum_i p_i \frac{\partial \pi_j(\mathbf{p})}{\partial p_i}$, where $0 < \lambda_j \leq 1$ is the degree of homogeneity

where $0 < \lambda_j \leq 1$ is the degree of homogeneity for production activity j.

Conditions (3.2) and (3.9) together with equation (3.11e) imply:

$$-y_j \pi_j(\mathbf{p}) = 0, \quad \forall j \tag{3.12}$$

This means that, in equilibrium, an activity that earns negative profit is non-active, enabling the modeling of non-profitable benchmark activities that may become active when a policy instrument is applied, e.g. the stimulation of an ineffective sector when the taxable sector is taxed. The reverse can also be modeled, of course, i.e. the decline of a previously profitable activity when a policy that disfavors it is introduced, as will be shown in one scenario addressed in this paper.

To summarize:

Conditions (3.2) and (3.7), and equation (3.12) reveal the complementarity relation between the profit function, $\pi(\mathbf{p})$, and the activity levels, $\mathbf{y}$:

$$-\pi_j(\mathbf{p}) \geq 0, \quad y_j \geq 0, \quad -y_j \pi_j(\mathbf{p}) = 0, \quad \forall j \tag{3.13}$$

From (3.3), (3.8), and (3.10), we see the complementarity relation between the excess supply function, $\xi(\mathbf{p}, \mathbf{y})$, and prices, $\mathbf{p}$:

$$\xi_i(\mathbf{p}, \mathbf{y}) \geq 0, \quad p_i \geq 0, \quad p_i \xi_i(\mathbf{p}, \mathbf{y}) = 0, \quad \forall i \tag{3.14}$$

Finally, identity (3.4) for income can be written in terms of complementarity to itself,

$$m_k - \sum_i p_i w_{i,k} = 0, \quad m_k \geq 0, \quad m_k(m_k - \sum_i p_i w_{i,k}) = 0, \quad \forall k \tag{3.15}$$

and constraints (3.5) and (3.6) can be stated in terms of complementarity to variables $\mathbf{u}$ and $\mathbf{p}^u$ according to:

$$e_k(\mathbf{p}) - p_k^u = 0, \quad u_k \geq 0, \quad u_k(e_k(\mathbf{p}) - p_k^u) = 0, \quad \forall k \tag{3.16}$$

$$u_k - \frac{m_k}{p_k^u} = 0, \quad p_k^u \geq 0, \quad p_k^u(u_k - \frac{m_k}{p_k^u}) = 0, \quad \forall k \tag{3.17}$$

Relations (3.13), (3.14), (3.15), (3.16), and (3.17) can be written more compactly as:

$$\begin{aligned}
-\boldsymbol{\pi}(\mathbf{p}) \geq \mathbf{0} \;&\perp\; \mathbf{y} \geq \mathbf{0} \\
\mathbf{e}(\mathbf{p}) - \mathbf{p^u} = \mathbf{0} \;&\perp\; \mathbf{u} \geq \mathbf{0} \\
\boldsymbol{\xi}(\mathbf{p}, \mathbf{y}) \geq \mathbf{0} \;&\perp\; \mathbf{p} \geq \mathbf{0} \\
\mathbf{u} - \frac{\mathbf{m}}{\mathbf{p}^u} = \mathbf{0} \;&\perp\; \mathbf{p}^u \geq \mathbf{0} \\
\mathbf{m} - \mathbf{W}\mathbf{p} = \mathbf{0} \;&\perp\; \mathbf{m} \geq \mathbf{0}
\end{aligned}$$

Which is equivalent to:

$$\mathbf{c}(\mathbf{p}) \geq \mathbf{r}(\mathbf{p}) \qquad \perp \ \mathbf{y} \geq \mathbf{0} \tag{3.18a}$$

$$\mathbf{e}(\mathbf{p}) = \mathbf{p}^u \qquad \perp \ \mathbf{u} \geq \mathbf{0} \tag{3.18b}$$

$$\mathbf{y}^T \nabla_p \mathbf{r}(\mathbf{p}) + \mathbf{1}^T \mathbf{W} \geq \mathbf{y}^T \nabla_p \mathbf{c}(\mathbf{p}) + \mathbf{m}^T \nabla_p \mathbf{e}(\mathbf{p}) \qquad \perp \ \mathbf{p} \geq \mathbf{0} \tag{3.18c}$$

$$\mathbf{u} = \frac{\mathbf{m}}{\mathbf{p}^u} \qquad \perp \ \mathbf{p}^u \geq \mathbf{0} \tag{3.18d}$$

$$\mathbf{m} - \mathbf{W}\mathbf{p} = \mathbf{0} \qquad \perp \ \mathbf{m} \geq \mathbf{0} \tag{3.18e}$$

where:

$\boldsymbol{\pi}(\mathbf{p})$ is the vector-valued (m x 1) unit profit function,

$\boldsymbol{\xi}(\mathbf{p}, \mathbf{y})$ denotes the vector-valued (n x 1) excess supply function,

$\boldsymbol{c}(\mathbf{p})$ indicates the vector-valued (m x 1) unit cost function,

$\boldsymbol{r}(\mathbf{p})$ signifies the vector-valued (m x 1) unit revenue function,

$\boldsymbol{e}(\mathbf{p})$ represents the vector-valued (h x 1) expenditure function, and

$\frac{\boldsymbol{m}}{\mathbf{p}^u}$ is defined as the per element division.

In equilibrium, commodities in excess supply must have zero prices, and where commodities have positive prices, markets must clear. An activity that earns negative economic profit is idle, and an activity with a positive activity level must earn zero economic profit. Further, negative prices, negative activity levels, negative excess supply, or positive economic profit cannot exist in equilibrium due to the restrictions $\mathbf{0} \leq \mathbf{p} < \infty$, on prices, and $\mathbf{0} \leq \mathbf{y} < \infty$ on activity levels.

4. The Ecological model

The ecological part of the model is a straightforward forest growth model describing the population dynamics of forests in which a clear-cut harvesting regime is applied. The model divides the forest's state, $\mathbf{s}_t$ at time period t, of an area of standing stock into compartments, $s_{t,a}$, in which all of the threes are of the same age class. The age-classes are ordered in increasing age, labeled a, with range $(0, 1, \ldots, A)$. Index 0 represents the bare land age-class, and index A the oldest age-class. The terms "compartment" and "age-class" will be used interchangeably in the paper. Inter-period growth is modeled by moving the standing area in each age-class $(s_{t,a})$, minus the harvested area (h_{t,a_h}), into the next age class, $(s_{t+1,a+1})$. An area is considered to be harvested, (h_{t,a_h}), just before the hosted forest enters a new state, and is restricted to a subset, $a_h \in (k, k+1, \ldots, A)$, of available age-classes.

The system of equations describing the growth and harvest of the forest areas are:

$$s_{t,0} = \sum_{a_h=k}^{A} h_{t,a_h} \tag{4.1a}$$

$$s_{t,1} = s_{t-1,0} \tag{4.1b}$$

$$\vdots$$

$$s_{t,k} = s_{t-1,k-1} - h_{t,k} \tag{4.1c}$$

$$\vdots$$

$$s_{t,A} = s_{t-1,A-1} + s_{t-1,A} - h_{t,A} \tag{4.1d}$$

Equation (4.1a) shows that the bare land compartment, $s_{t,0}$, receives input from all areas harvested. It also implies that compartments from index k up to the last age-class A may be subject to harvest. Equations (4.1b) up to, but not including, equation (4.1c) show how area compartments of all age-classes for which harvest is not allowed move up to the next compartment, while equations (4.1c) up to, and including (4.1d) describe the growth of the compartments that are harvested. Equation (4.1d) imposes the model's upper limit on the number of age-classes, and growth in the oldest compartment, A, is accounted for by the additional term "$s_{t-1,A}$" in equation (4.1d).

The above system of equations can be written more compactly in matrix form:

$$\mathbf{s}_t = \mathbf{G}\left(\mathbf{s}_{t-1} + \bar{\mathbf{s}}_0\right) - (\mathbf{I}^* - \mathbf{B})\mathbf{h}_t \tag{4.2}$$

where $\bar{\mathbf{s}}_0$ is the initial forest state entering the model at time period 0. $\mathbf{G}$

is a matrix responsible for shuffling previous compartments into corresponding present age-class. Matrix $\mathbf{I}^*$, is a reduced identity matrix with ones in positions where harvest is allowed, $\mathbf{B}$ is a matrix that projects the harvest vector down to the bare land compartment (ones in the first row and zeros otherwise), and the harvest vector $\mathbf{h}_t$ will have zeros up to index k. For example, in a system with four compartments (0-3) and harvest allowed in age-classes 2 and 3, these matrices would have the form:

$$\mathbf{I}^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathbf{G} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The biomass volume entering the industrial sectors using the harvested biomass is simply represented by a vector describing the biomass volume per unit area for all age-classes. This way of modeling the forest growth and harvest allows a heterogeneous perspective of the biomass use, where the age-structure of the biomass may represent diversified characteristics in demand. The model developed in this paper includes 16 age-classes, and harvest is allowed in the ten oldest compartments (although in practice, some younger biomass that could be used for some purposes may be harvested in thinnings). The growth of a representative tree at different ages can be seen in Figure (1). The growth starts to decline at the age 60 years which is chosen as the youngest allowable age for harvest.

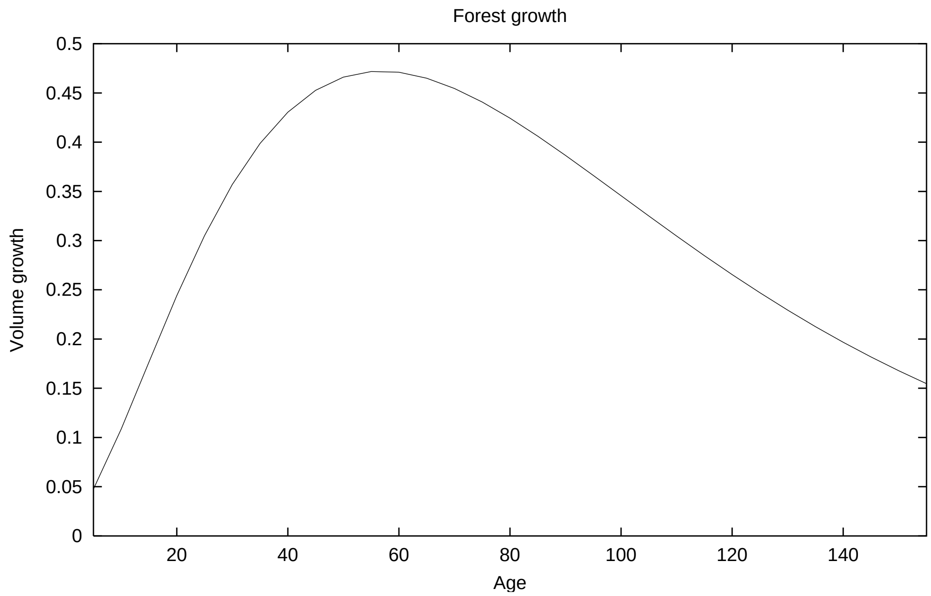

Figure 1: Volume growth of an average tree as a function of age.

5. The combined Ecol-Econ model

The ultimate purpose of this paper is to provide a model framework capable of describing the long-term changes to forest harvest behavior arising in response to policy interference. Effects of policies on other sectors that are heavily dependent on forest biomass, such as the pulp, wood, and (at least in Sweden) biomass incineration for energy production sectors, are also specifically considered. For simplicity and to focus sharply on the impact of biomass dynamics on behavioral changes, investment in man-made capital stock and growth of labor force are disregarded. The dynamics of the presented model are entirely governed by the growth and harvest of the forest biomass stock.

In a deterministic setting, dynamic modeling requires an assumption regarding the degree of foresight of economic agents. In this model setup, agents' expectations of future prices correspond to realized future prices in the simulation, in other words, agents are assumed to have perfect foresight. The notion of perfect foresight is coupled with the assumption of infinitely-lived agents who make choices between the consumption levels of current and future generations. The representative agents are assumed to maximize welfare subject to an inter-temporal budget constraint, which is equivalent to minimizing inter-temporal expenditure subject to restrictions on achievable welfare.

In this section a compact set of variables and functionals are presented to provide an overview of the model, but due to the multitude of details related to biomass-related activities the model's finer details and functional forms are presented separately in Appendix A.

5.1. Model variables

Two classes of agents are included in the model setup, forest owners (FO), and the owners of capital and labor (KLO). Associated with these agents are the variables for income levels:

- m_{FO} denotes income earned from roundwood by household FO,
- m_{KLO} represents income from factors labor L and capital K, earned by household KLO.

The model considers four top-level intra-temporal production activities producing one composite rest-of-industry macro good, and three forest biomass related goods (sawn wood, pulp, and bio-energy). The four top-level goods are aggregated into an intra-temporal composite consumption good that the agents FO and KLO demand. All production activities are modeled under the assumption of constant return to scale (CRTS). The activity levels of these intra-temporal production activities are:

$\mathbf{x}_t^f$ signifies a vector of activity levels for each of the three biomass production activities, where $(i, p, w) \in f$ and i stands for energy production by incineration, p for pulp and paper production, and w for wood related product productions,

x_t^o represents the activity level for an aggregated macro good,

x_t denotes an aggregated consumption good indicating the per period consumption.

The demands of the aggregated consumption good by FO and KLO are determined under the assumption of minimization of lifetime expenditure subject to some achievable utility, that is, maximizing life time utility subject to a life time budget restriction.

u^{FO} represents a lifetime utility index for the representative forest owner FO,

u^{KLO} is the lifetime utility index for the representative owner of capital and labor, KLO.

The forest area distribution vector, areas harvested and roundwood harvested are represented by:

$\mathbf{s}_t$ indicating the forest area distribution over age classes,

$\mathbf{h}_t$ signifying the forest area harvested over age classes,

$\mathbf{b}_t$ denoting the volume of roundwood harvested over age classes,

The set of market prices that are included in the model are:

$\mathbf{p}_t^{x_f}$ the price vector of the three intra-period top level biomass goods,

$p_t^{x_o}$ the price of the intra-period top level macro good,

p_t^{x} the intra-period price of the consumption composite,

$\mathbf{p}_t^{s}$ the vector of land prices for age classes,

$\mathbf{p}_t^{b}$ the vector of prices for roundwood harvested in different age classes,

p_t^{L} the price of labor,

p_t^{K} the rental price of capital, and

p_u^{FO}, p_u^{KLO} the prices of one unit of lifetime utility for agent FO and KLO respectively.

Further variables used in the detailed disentanglement of age-structured biomass are presented in Appendix A.

5.2. Equilibrium conditions

Cost, expenditure, and revenue functions are all modeled as constant elasticity of substitution (CES) and constant elasticity of transformation (CET) functions (with forms presented in Appendix A.3).

Top-level biomass goods are produced from roundwood, capital and labor. Note that various sub- activities, -prices, and -functions are included within this specification (details can be found in Appendix A). The zero-profit conditions for the three top-level biomass production processes, using capital, labor, and age-structured roundwood, are specified as:

$$\mathbf{c}_t^{x_f}(p_t^K, p_t^L, \mathbf{p}_t^b) \geq \mathbf{p}_t^{x_f} \qquad \perp \qquad \mathbf{x}_t^f \geq 0 \tag{5.1}$$

where $\mathbf{c}_t^{x_f}(\cdot)$ is the vector-valued cost function for the three top-level biomass production activities.

The top-level macro good is produced using capital and labor, and is controlled by the complementary zero profit condition:

$$c_t^{x_o}(p_t^K, p_t^L) \geq p_t^{x_o} \qquad \perp \qquad x_t^o \geq 0 \tag{5.2}$$

where $c_t^{x_o}(\cdot)$ is the cost function for the top-level production activity of the macro good.

The production of the aggregated consumption good is controlled by the relation:

$$c_t^x(\mathbf{p}_t^{x_f}, p_t^{x_o}) \geq p_t^x \qquad \perp \qquad x_t \geq 0 \tag{5.3}$$

where $c_t^x(\cdot)$ is the cost function for the aggregated consumption good.

The complementary constraints on life-time consumption for the two households are defined as:

$$e^{FO}(\mathbf{p}^x) \geq p_u^{FO} \qquad \perp \qquad u^{FO} \geq 0 \tag{5.4}$$

$$e^{KLO}(\mathbf{p}^x) \geq p_u^{KLO} \qquad \perp \qquad u^{KLO} \geq 0 \tag{5.5}$$

where $e^{FO}(\cdot)$ and $e^{KLO}(\cdot)$ are the lifetime expenditure functions for forest owners and capital-labor owners, respectively.

The inter-temporal unit lifetime expenditure functions, $e^{FO}(\cdot)$ and $e^{KLO}(\cdot)$, arise from behavior choices consistent with expenditure-minimization of infinitely-lived representative agents,

$$\min_{x_t} \sum_t^{\infty} p_t^x x_t \tag{5.6}$$

subject to unit lifetime utility:

$$\Upsilon(\mathbf{x}) = \sum_t^{\infty} \left[\left(\frac{1}{1+\rho} \right)^t \upsilon(x_t) \right] = 1 \tag{5.7}$$

where ρ is the rate of time preference and $\upsilon(\cdot)$ is the instantaneous utility function. The lifetime utility function, $\Upsilon(\cdot)$, is assumed to be isoelastic and additively separable over time. With isoelastic lifetime utility the instantaneous utility function is defined as:

$$\upsilon(x_t) = \frac{x_t^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \tag{5.8}$$

where η is the constant inter-temporal elasticity of substitution in consumption. Assuming additively separable lifetime utility, and that the system has reached steady state in period T, the above minimization problem can be rewritten as,

$$\min_{x_t} \sum_{t}^{T} p_t^x x_t + C_1 \tag{5.9}$$

subject to:

$$\Upsilon(\mathbf{x}) = \sum_{t}^{T} \left[\left(\frac{1}{1+\rho} \right)^t \upsilon(x_t) \right] = 1 - C_2 \tag{5.10}$$

where $C_1 = \sum_{T+1}^{\infty} p_t^x x_t$ and $C_2 = \sum_{T+1}^{\infty} \left[\left(\frac{1}{1+\rho} \right)^t \upsilon(x_t) \right]$ are some constants. This problem in turn, can be rewritten as,

$$\mathbf{e}_k(\mathbf{p}) = \min_{x_t} \sum_{t}^{T} p_t^x x_t \tag{5.11}$$

subject to unit linear homogeneous lifetime utility:

$$\Gamma_k\left(\Upsilon_k(\mathbf{x})\right) = 1 \tag{5.12}$$

where $\Gamma_k(\cdot)$ denotes a homothetic transformation and $k \in \{KLO, FO\}$ is used as an index separating the two representative households.

If prices for factors and goods are determined in competitive markets, balance of supply and demand is assured. Using Shepard's lemma, we can derive supply and compensated demand functions of goods and factors (see complementary relations (3.18b) and (3.18c)). Market clearance conditions for this stylized economy then read as:

$$K_t \geq \mathbf{x}_t^{f\prime} \nabla_{p^K} \mathbf{c}_t^{x_f}(\cdot) + x_t^o \frac{\partial c_t^{x_o}(\cdot)}{\partial p_t^K} \qquad \perp \qquad p_t^K \geq 0 \tag{5.13}$$

$$L_t \geq \mathbf{x}_t^{f\prime} \nabla_{p^L} \mathbf{c}_t^{x_f}(\cdot) + x_t^o \frac{\partial c_t^{x_o}(\cdot)}{\partial p_t^K} \qquad \perp \qquad p_t^L \geq 0 \tag{5.14}$$

$$\mathbf{x}_t^f \geq x_t \nabla_f c_t^x(\cdot) \qquad \perp \qquad \mathbf{p}_t^{x_f} \geq \mathbf{0} \tag{5.15}$$

$$x_t^o \geq x_t \frac{\partial c_t^x(\cdot)}{\partial p_t^{x_o}} \qquad \perp \qquad p_t^{x_o} \geq \mathbf{0} \tag{5.16}$$

$$\mathbf{x} \geq u^{FO} \nabla e^{FO}(\mathbf{p}^x) + u^{KLO} \nabla e^{KLO}(\mathbf{p}^x) \qquad \perp \qquad \mathbf{p}^x \geq \mathbf{0} \tag{5.17}$$

$$u^{FO} \geq \frac{m_{FO}}{p_u^{FO}} \qquad \perp \qquad p_u^{FO} \geq 0 \tag{5.18}$$

$$u^{KLO} \geq \frac{m_{KLO}}{p_u^{KLO}} \qquad \perp \qquad p_u^{KLO} \geq 0 \tag{5.19}$$

5.3. Ecological equilibrium conditions

From equation (4.1) it is apparent that growth is represented as a shift of forest areas from one age class to the next, and harvesting moves areas from different age classes to the bare land age class. The zero-profit conditions determine inter-temporal forest growth and harvest, as follows. Firstly, in equilibrium, the market value of each unit area of forest in age-class a purchased at the beginning of period t can be no less than the value of the unit area $a+1$ to which it will grow at the start of the subsequent period. This translates to the inter-temporal "zero-profit" condition for forest growth:

$$\mathbf{p}_t^s \geq \mathbf{G}'\mathbf{p}_{t+1}^s \qquad \perp \qquad \mathbf{s}_t \geq \mathbf{0}, \quad (t < T) \tag{5.20}$$

Secondly, the opportunity to make revenue by harvesting age-classes a_h limits the market price of forest land in age-classes a_h and bare land age class, a_0. The bare land age-class is the age class to which the harvested area unit will move. In other words, the unit revenue for harvesting an area unit of age a_h, is the price of a unit volume of roundwood at age a_h provided from that area unit plus the price of a unit bare land area. The opportunity cost of this revenue is the price of the forest land if it had not been harvested. This gives the intra-temporal zero-profit condition for harvest:

$$\mathbf{p}_t^s \geq \mathbf{V}'\mathbf{p}_t^b + p_{t,0}^s \qquad \perp \qquad \mathbf{h}_t \geq \mathbf{0} \tag{5.21}$$

where $p_{t,0}^s$ is the bare land price.

From complementary relation 5.20, the terminal price vector, $\mathbf{p}_T^s$, is clearly unconstrained because $\mathbf{p}_{T+1}^s$ is beyond the time-horizon of the model. If the time-horizon is taken to be sufficiently long the forest is assumed to have reached steady state. In steady state, quantitative variables all remain the same over the modeled time periods and price variables change according to discounting (following the discrete version of transversality conditions commonly used in optimal control theory). The terminal conditions of variables $\mathbf{p}_T^s$ are therefore taken to represent the system when it has reached steady state. It is stated here as:

$$(1+\rho_{FO})\mathbf{p}_T^s \geq \mathbf{p}_{T-1}^s \qquad \perp \qquad \mathbf{s}_T \geq \mathbf{0} \tag{5.22}$$

where ρ_{FO} is the rate of time preference of the forest.

The ecological growth equation limits the price of forest land resulting in a market clearance condition for growth:

$$\mathbf{s}_t = \mathbf{G}\mathbf{s}_{t-1} - (\mathbf{I}-\mathbf{B})\mathbf{h}_t - \bar{\mathbf{s}}_0 \qquad \perp \qquad \mathbf{p}_t^s \geq 0 \tag{5.23}$$

The volume of roundwood harvested in each age class is simply modeled by translating the harvested area measure to a volume roundwood measure via the diagonal volume per area matrix, hence market clearance conditions for the volume of roundwood harvested is stated as:

$$\mathbf{b}_t = \mathbf{V}\mathbf{h}_t \qquad \perp \qquad \mathbf{p}_t^b \geq 0 \tag{5.24}$$

5.4. Income balance

Income for the forest owner, FO, is defined by the sum of present value flow of roundwood into the economy. An equivalent definition would be to use the land value of the first period minus the land value of the last period, but the former is chosen for computational convenience:

$$m_{FO} = \sum_t \mathbf{b}'_t \mathbf{p}^b_t \qquad \perp \qquad m_{FO} \geq 0 \tag{5.25}$$

Income for the capital-labor owners, KLO, is calculated as the sum of the present value of the fixed stocks of capital and labor:

$$m_{KLO} = \sum_t K_t p^K_t + L_t p^L_t \qquad \perp \qquad m_{KLO} \geq 0 \tag{5.26}$$

6. Simulations

The role of forestry in climate policy has been extensively studied from an economic perspective (see Sedjo et al. (1995) or Parks et al. (1997) for summaries of the economic aspects, and Sedjo et al. (1997) for summaries of the relevant literature from a broader perspective). Forest ecosystems constitute large carbon pools and could have significant impacts on changes in the carbon dioxide (CO_2) concentration in the atmosphere (Birdsey (1992), Dixon et al. (1994),Houghton (1991), Winjum et al. (1992)). Thus, adding forest sequestration as an option to a country's climate policy can offer significant net benefits, although these benefits are likely to vary considerably across countries.

Sequestration of CO_2, has attracted growing interest among government policy makers recently. For example, the Swedish government bill Government Bill 2007/08:108, (2008), states that it is *important to analyze the conditions for the instruments and controls eligible for the forestry sector to contribute further to a cost-effective achievement of Swedish climate policy.*

Technically, there are several ways to increase the CO_2 sequestration of forests, such as increasing the forest area, changing management regimes for existing forests, prolonging the lifetime of timber products and increasing the utilization of bio-energy. Of course, forests sequestrate CO_2 even if they are not managed for this purpose. However, CO_2 sequestration is not completely compatible with all the other objectives of forest management. Thus, optimizing the utilization of forest resources to increase CO_2 sequestration may require adjustment of forestry practices. This paper focuses on the management of existing forests, particularly on the incentive schemes that could be applied to increase CO_2 sequestration by forest owners. The policy instruments considered can be described as follows.

Scenario 1 Awards of payments to forest owners for CO_2 sequestrated due to forest growth in each time period, and deductions of tax for the CO_2 content of each harvest.

Scenario 2 In addition to the measures applied in Scenario 1, the provision of rebates for forest owners on CO_2 payments related to harvest, depending on the final products the harvested roundwood biomass is used to make, and how long these products prevent the sequestrated CO_2 from entering the atmosphere.

Scenario 3 In addition to the measures applied in Scenarios 1 and 2, provision of rebates for forest owners on CO_2 payments related to harvest, depending on stumpage left in the forest and how long the stumpage prevents the sequestrated CO_2 from entering the atmosphere.

Effects of similar types of policy instrument have been investigated in other studies, see for example Van Kooten et al. (1995) and Gong and Kriström (1999). The main difference here is in the heterogenity of the biomass.

Three additional types of variables must be included to accommodate the scenarios outlined above: a variable for each period indicating the quantity of sequestrated CO_2; a variable for tax income level associated with an introduced intermediary agent (GOVT), initially without endowments, redistributing taxes and subsidies; and an endogenous tax levied on consumption introduced to compensate for cases when the CO_2 payment associated with harvest is insufficient to cover expenditure on CO_2 sequestration. In addition to these three variables a fixed CO_2 price vector (a parameter) is needed:

m_{GOVT}	the tax income for GOVT,
n^{tax}	endogenous tax rate,
q_t^+	quantity of positive CO_2 flux in period t,
q_t^-	quantity of negative CO_2 flux in period t,
$p_t^{CO_2}$	(discounted) price parameter of CO_2.

The tax income variable is the sum of the endogenous ad valorem tax on consumption and the tax payments on negative CO_2 flux:

$$m_{GOVT} = n^{tax} \sum_t x_t p_t^x + \sum_t q_t^- p_t^{CO_2} \qquad \perp \qquad m_{GOVT} \geq 0$$

The endogenous tax variable is controlled by the equality of tax income and expenditure on CO_2 subsidies. Note that if the tax income is greater than expenditure, the endogenous tax serves as a consumption subsidy. This is achieved by defining the variable n^{tax} as free, meaning that its lower bound is set to $-\infty$ and that the controlling relation can be effectively described by the equation:

$$m_{GOVT} = \sum_t q_t^+ p_t^{CO_2} \qquad \perp \qquad n^{tax}$$

The endogenous tax is levied on consumption and therefore enters the zero-profit condition for the aggregated macro good, which has to be altered accordingly:

$$c_t^x(\mathbf{p}_t^{x_f}, p_t^{x_o})(1 + n_t^{tax}) \geq p_t^x \qquad \perp \qquad x_t \geq 0$$

The positive CO_2 flux, q_t^+, in each period is calculated as the change in forest volume during that period plus the quantitative tax rebate equivalent, which is conditional on biomass use and is modeled as a CO_2 byproduct of the biomass production activities detailed in Appendix A. This way of modeling the tax rebate means that the forest owner receives the rebate via a higher price of roundwood depending on where it is used. The CO_2 flux is the base for which the forest owners receive a subsidy, and is defined by:

$$q_t^+ = \mu \mathbf{dv}'(\mathbf{s}_{t-1} + \bar{\mathbf{s}}_0) + \kappa_p \mathbf{b}_{t,a_p} + \kappa_w \mathbf{b}_{t,a_w} \qquad \perp \qquad q_t^+ \geq 0$$

where μ is the CO_2 content of biomass, the elements of $\mathbf{dv}$ are $dv_a = v_{a+1} - v_a$, and κ_p and κ_w, are the tax rebate parameters associated with CO_2 decay of biomass in the final products pulp or sawn wood[4], and $\mathbf{b}_{t,a_p}$ and $\mathbf{b}_{t,a_w}$ are the amounts of biomass used in the respective products (see Appendix A for

[4]The factors are calculated based on the half-life of the products CO_2 content $\kappa_p = \theta_p^{BIO} \sum_{\tau=0}^{\infty} \left(\frac{1}{1+\rho}\right)^{\tau} \left(\frac{1}{2}\right)^{\frac{\tau}{h_p}}$, where θ_p^{BIO} is the share of biomass used in production that is present in the final pulp product, and h_p is the half-life of CO_2 contained in pulp products. Similar calculations are applied for sawn wood.

definitions of $\mathbf{b}_{t,a_p}$ and $\mathbf{b}_{t,a_w}$).

The negative CO_2 flux in each period for which the forest owner pays tax, is calculated as the CO_2 content of harvest minus the rebate for CO_2 decay of stumpage left in the forest:

$$q_t^- = \mu(1 - \kappa_{stump})\mathbf{h}'\mathbf{V}\mathbf{p}^{CO_2} \qquad \perp \qquad q_t^- \geq 0$$

where κ_{stump} is the CO_2 tax rebate parameter associated with stumpage CO_2 decay.

The introduced variables and conditions alter some of the previously defined constraints. The zero-profit condition on forest growth determining price relations for forest land now also accounts for the additional values associated with CO_2 sequestration. In equilibrium, the market land value of each unit area of forest in age-class a purchased at the beginning of period t can be no less than the value of the unit area $a+1$ to which it will grow at the start of the subsequent period plus the value of CO_2 sequestrated by that area unit during period t. This translates to an extended inter-temporal "zero-profit" condition for forest growth in the scenarios:

$$\mathbf{p}_t^s \geq \mathbf{G}'\mathbf{p}_{t+1}^s + \mu p_{t+1}^{CO_2}\mathbf{dv} \qquad \perp \qquad \mathbf{s}_t \geq \mathbf{0}, \qquad (t < T)$$

The terminal forest growth condition remains the same because $(1+\rho)p_T^{CO_2} = p_{T-1}^{CO_2}$, by definition.

The harvesting constraints also change. The land value per unit harvested area can be no less than the revenue in terms of the roundwood value plus bare land value of that area unit harvested minus the value of negative CO_2 flux per unit harvested area. The value of positive CO_2 flux has to be subtracted from the land value while it enters as positive income for the forest owner. The extended intra-temporal "zero-profit" condition for harvest in the scenarios reads:

$$\mathbf{p}_t^s \geq \mathbf{V}\mathbf{p}_t^b + p_{t,0}^s - \mu(1-\kappa_{stump})p_t^{CO_2}\mathbf{V} \qquad \perp \qquad \mathbf{h}_t \geq \mathbf{0}$$

Finally, the definition of the forest owner's income requires adjustment due to income from positive CO_2 flux while the forest grows and the tax payment for losses of sequestrated CO_2 while harvesting:

$$m_{FO} = \mathbf{b}'\mathbf{p}^b + \mathbf{q}'\mathbf{p}^{CO_2} - \mathbf{q}^{-\prime}\mathbf{p}^{CO_2} \qquad \perp \qquad m_{FO} \geq 0$$

Note that the tax rebate accrues to the forest owner in terms of a higher price received on roundwood sold.

Essential features of the MCP modeling format are weak inequalities and complementary slackness. This enables the modeling of problems where, for at least some of these inequalities, we do not know a priori, which will hold as strict inequalities and which will hold with equality at an equilibrium.

Two classes of formulations may be applied in such a model. If a commodity can be produced by alternative technical processes, as seen here when producing biomass for input to the incinerating industry, inequalities arise. The biomass product might stem from residuals from the pulp or wood industries, or as roundwood harvested from the forest. If the price of harvested roundwood for incineration is too high, this activity becomes inoperative, and inputs will depend on biomass residuals from the wood and pulp industries. Second, the economic problem under consideration may involve various types of institutional constraints on prices or quantities. There can be upper or lower bounds on some prices, as in a fixed price model, or several single-period balance of payments constraints, as in a multiple-period model. The fixed price for CO_2 in this model effectively puts a lower bound on the industrial price of roundwood.

Another aspect of analyzing an economic equilibrium in the MCP format is the underlying assumption that the economic agents engage in individual optimization and the equilibrium outcome is determined by the interaction among these agents rather than by optimizing some single objective. In this model the separation of consumers into two groups, capital-labor owners and forest owners, enables investigations of welfare transfers between these two groups.

6.1. Numerical simulations

Artificial data were used in the simulations presented here, and the initial state of area distribution of the forest is one in which the forest is growing. The state of the forest is divided into 32 age classes, with bare land being one compartment. Harvest is allowed in the 16 oldest age classes. In the following figures the time horizon is truncated to 350 years. The time horizon of the model is 450 years with a time increment of 5 years, hence there are 90 distinct time periods. The time increment of 5 years and division of the forest into 32 equidistant age classes means that the oldest considered forest is 160 years old.

The parameters of interest in the simulations are the price of CO_2, $p_t^{CO_2}$, the tax rebate for roundwood used for pulp products, κ_p, the tax rebate for roundwood used for wood products, κ_w, and the tax rebate for stumpage left in forest, κ_{stump}.

The price per tonne CO_2, $p_t^{CO_2}$, is chosen such that the first period income shares in Scenario (1) would roughly match a price ratio of $\frac{p^{CO_2}}{p^{bio}} = \frac{15}{34}$ where p^{bio} is the price (not included in the model) of aggregated roundwood per m^3. The price of 15 EUR per tonne CO_2 is close to the price for tradable carbon permits on the European spot market during 2010, and 34 EUR per m^3 roughly matches the mean Swedish market price of timber. The price of CO_2 is then discounted by the discount rate, chosen to be 2 %, in subsequent periods to yield the full price path $p_t^{CO_2}$. For simplicity the same price path $p_t^{CO_2}$ is used in all scenarios, but could of course be changed for counterfactual analysis.

The tax rebate factors κ_p and κ_w used in Scenarios 2 and 3 are chosen to be 0.5 and 0.8 respectively. The tax rebate factor κ_{stump} used in scenario 3 is chosen to be 0.6. These tax rebate factors are chosen arbitrarily.

The three scenarios are designed to make forest owners delay harvest in order to induce a greater sequestration of CO_2. The resulting older forest contains more biomass which also mean more carbon dioxide (or, more strictly, carbon).

The model was implemented in MPSGE (Rutherford, 1999) as a subsystem of GAMS (Brooke et al., 1996) using PATH (Dirkse and Ferris, 1995) for solving the MCP problem. Though not explicitly necessary when using PATH, the first period price of aggregated consumption, p_0^x, was chosen as a numeraire[5].

6.2. Changes in growth, harvest, and carbon sequestration

Figure 2 shows how the average age changes in the benchmark and simulated scenarios. The *average age* of the forest is calculated by summing the area of specific age classes weighted by the area of standing stock they include, and dividing by the total area. The increases in *average age* at the beginning of the benchmark path indicates that the forest is initially growing;

[5]Choosing a numeraire relates to the closure of the model and involves fixing one variable, usually a price variable, at some level in order to make the system solvable. A side benefit of using a numeraire is that all changes in other variables in the counterfactual runs of the model, relate to this fixed variable.

that is, growth exceeds the amounts of biomass harvested. The increases in *average age* during the initial years is a result of the initial area distribution[6] of the standing stock and the system of demand for biomass that governs harvest behavior. After this initial rise, the average age levels out to that of the so-called *normal forest* which represents the steady state situation that holds when harvest equals growth and no further change occurs in the state of the forest between periods.

When forest owners are compensated for forest growth and taxed for harvest, the forest owners delay harvest, resulting in an older forest, as seen throughout the path of Scenario 1.

In Scenario 2 it is assumed that wood products have a longer "CO_2-life" than pulp products. This means that the two tax rebate factors κ_p and κ_w are related according to $\kappa_w > \kappa_p$. Further, production of wood products is assumed to rely more heavily than pulp production on roundwood from older forest. These two assumptions effectively translate into a higher tax rebate for harvests of older forest, which is manifested as an even greater increase in the average forest age in Scenario 2 than in Scenario 1.

[6]The initial area distribution is constructed such that the younger compartments of the forest are fed with a larger share of the total area available compared to what they would have, had the forest been in a steady state (normal forest).

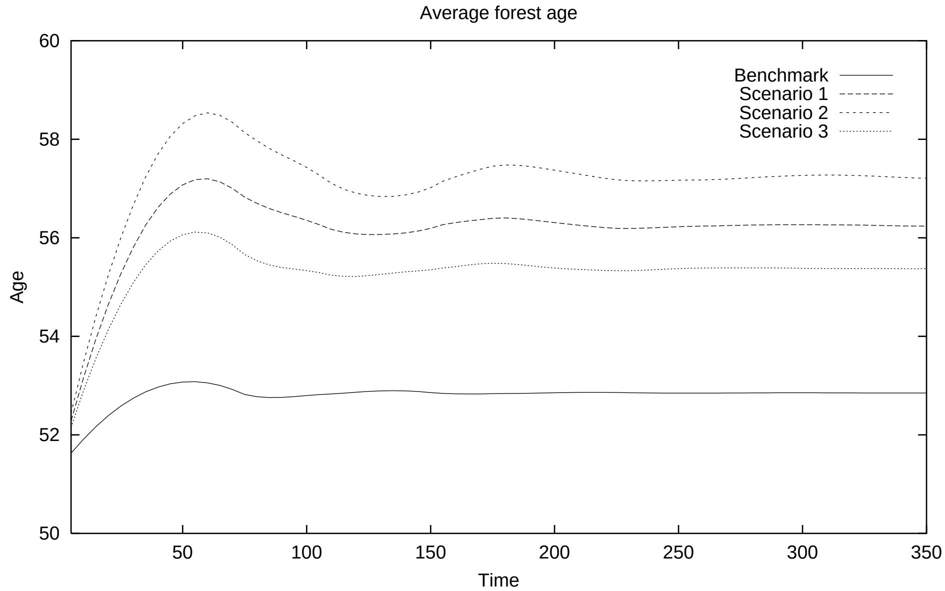

Figure 2: Changes in average age of the forest, based on the area distribution of the forest in different age classes. The benchmark path shows that, initially, growth exceeds amounts of biomass harvested, and the forest grows older. Policy measures applied in Scenario 1 induce forest owners to delay harvest which is manifested in an older forest, on average, than in the benchmark scenario. In Scenario 2, reduced tax on the use of older forest increases the average forest age even further. Tax rebates on stumpage left in the forest, introduced in Scenario 3, pulls harvesting towards forest of ages where the biomass growth is highest, and the average forest age is lower than the Scenarios 1 and 2.

The additional tax rebate introduced in Scenario 3, on stumpage left in the forest, is applied regardless of the age of the harvested forest. This type of rebate tends to shift the harvest towards forest of ages where the biomass growth is highest. The reason for this is that the forest owners receives the full subsidy for growth which pulls harvesting toward ages at which growth is high, but the burden of tax payments related to harvest is reduced irrespective of forest age, thus shortening harvest delays. As can be seen in Figure 1, the maximum growth period for this forest is around 60 years, which pulls the average age down in Scenario 3 compared to Scenarios 1 and 2.

Van Kooten et al. (1995) analyze the effects on optimal rotation age, for a single plot in a clear-cutting forest plantation, of compensating its owner for the CO_2 sequestrated, i.e. growth, and taxed for harvest. They also introduce a tax rebate parameter similar to the stumpage tax rebate parameter in this study. To compare the results of the present and cited studies, an *average harvest age* statistic is computed in a similar fashion to the average forest age statistic, based on the area-weighted ages of the harvested compartments. The average harvest age can be viewed as an aggregated version of the optimal harvest age calculated by Van Kooten et al. (1995). The time course of this statistic in each of the scenarios is presented in Figure 3. Turning our attention to the benchmark path it can be seen that the average harvest age is almost constant for the first 75 years and then lightly increases. This indicates that the forest is initially heavily populated by stands of young ages, which cannot be harvested and subsequently grow to ages at which harvest is allowed. When these younger forest areas grow even older they contribute to the increase in the average harvest age, as seen in the "bump" of the benchmark path. There is a small rebound at year 150, followed by an almost constant average harvest age in subsequent periods.

In the path of Scenario 1 (Figure 3), the constant part of the benchmark path is replaced by a fairly constant increase in the average harvest age up to 75 years, which marks the start of a "boom generation", when the average harvest age accelerates. Harvest is delayed here in response to the growth subsidies and harvest taxes. During subsequent periods the general pattern is the same as in the benchmark path, but more accentuated. Both the peak and the rebound are relatively greater than in the benchmark case. In Scenario 2, as expected from the average forest age trends (Figure 2), the initial increase and subsequent decline in average forest age are even sharper. In Scenario 3, the average harvest age is younger than in Scenarios 1 and 2 throughout the entire modeled period, in accordance with simulated results of reductions in tax payments reported by Van Kooten et al. (1995). Thus, although Van Kooten et al. (1995) focused on a specific plot, while this study considers the entire area available for timber production, the results are comparable.

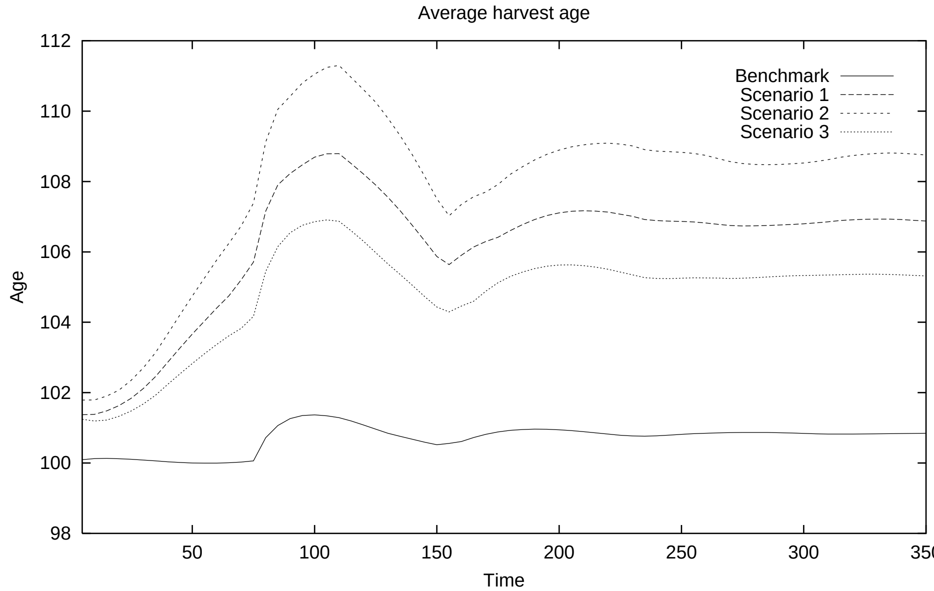

Figure 3: Changes in average harvest age in the benchmark and other scenarios. The benchmark path shows that on a relatively large proportion of the forested area the initially young trees age, thereby increasing the average harvest age. Paths of scenarios 1, 2, and 3 all display the effects of harvest delays induced by the introduced policy measures. Scenario 2, which includes a tax rebate targeting CO_2 storage in products, result in the largest increase in average harvest age. In Scenario 3, the effect of tax rebates dependent on the CO_2 content of stumpage left in the forest dampens the harvest postponement relative to Scenarios 1 and 2.

CO_2 contained in the forest, and pulp and wood forest products, is represented by a biomass CO_2 stock index. The CO_2 stock index is calculated as the CO_2 content of the forest stock plus CO_2 captured in the forest products *pulp* and *wood* which are assumed to contain CO_2 with half-lives of 13 and 30 years, respectively, described as medium-short and medium-long in the study by Pussinen et al. (1997). The main rationale for this is that forest products produced mainly from old forest retain CO_2 longer than those from younger forests, and products made from wood store carbon for longer than pulp-based paper. Incineration is considered to release the carbon immediately. However, full consideration of the effects of incineration might call for some more detailed analysis since unused biomass, i.e. tops and branches, that are not suitable for pulp and wood product manufacture, would not release carbon dioxide directly, but rather during a period of time when the biomass decomposes. In this model all forest biomass is used in the activities of growth, incineration, pulp production, and wood production. The calculations of the stored CO_2 index are shown below:

$$
\begin{aligned}
\phi_s(t) &= \boldsymbol{\mu}'\mathbf{s}_t \\
\phi_p(t) &= \sum_{\tau=0}^{t} \left[\theta_p^{BIO} \boldsymbol{\mu}'\mathbf{h}_{\tau,a_p} \left(\frac{1}{2}\right)^{\frac{t-\tau}{h_p}} \right] \\
\phi_w(t) &= \sum_{\tau=0}^{t} \left[\theta_w^{BIO} \boldsymbol{\mu}'\mathbf{h}_{\tau,a_p} \left(\frac{1}{2}\right)^{\frac{t-\tau}{hw}} \right] \\
\phi(t) &= \phi_s(t) + \phi_p(t) + \phi_w(t)
\end{aligned}
$$

where:

θ_p, θ_w	are the shares of used biomass that end up in pulp and wood final products respectively,
h_p, h_w	are the half-life for the specific products (pulp and wood, respectively)
$\phi_p(t), \phi_w(t)$	denotes the carbon dioxide still contained in pulp and wood products,respectively, at the beginning of time period t,
$\phi_s(t)$	is carbon dioxide contained in the forest at time period t,
$\phi(t)$	is total carbon dioxide contained in the forest and forest products in time period t.

Figure 4 shows time courses of the amount of CO_2 sequestered in forest biomass and products in each of the three scenarios, relative to benchmark (business as usual) scenario levels, illustrating the responses in this respect to the simulated policy measures. The paths show that the sequestered CO_2 is maximal at around year 75 for all scenarios, indicating that effects of the applied policy proposals are delayed, due to the age structure and growth capability constraints on the biomass stock. For Scenario 1 the maximal additional CO_2, relative to the amount that would have been sequestered with no policy interference, is about 14 %, and the steady state increase is slightly more than 12.5 %. The greatest increase of the CO_2 stock is provided by Scenario 2, showing the benefits of tax reductions constructed to favor older forest and the manufacture of products that release carbon slowly. Here the maximum additional amount and additional steady state equilibrium amount, relative to benchmark levels, are about 18 % , and slightly more than 16 % respectively. The lowest additions and, as shown below, most costly outcomes are those from Scenario 3, which results in maximal and steady state increases of 10 %, and slightly more than 9.5 %, respectively.

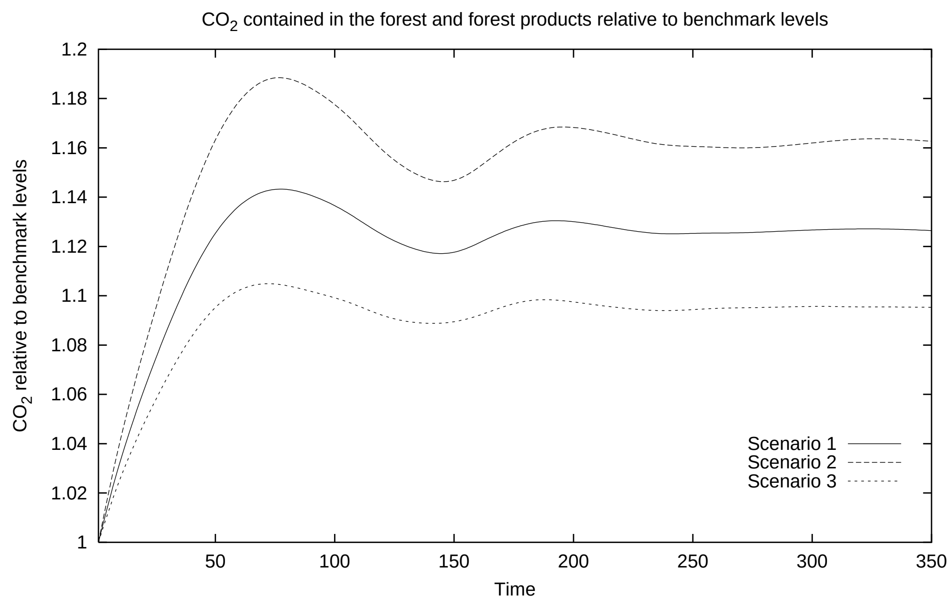

Figure 4: Carbon dioxide contained in forest biomass and products, relative to benchmark scenario levels. The CO_2 contained is maximal at around year 75 for all scenarios, illustrating the fact that effects of introducing policies take time to establish. In Scenario 1 the maximum and steady state additional amounts of CO_2 relative to benchmark levels are about 14 % and 12.5 %, respectively. Scenario 2 provides the greatest increases in the CO_2 stock (maximum and steady state increases of 18 % and 16 %, respectively). The lowest increases, and most costly results, are those from Scenario 3 (maximum and steady state increases of 10 % and 9.5 %, respectively).

6.3. Changes in biomass use

The simulations based on the three scenarios suggest that shares of harvested biomass used in the incineration, pulp, and wood sectors are likely to change. Figures 5, 6, 7, and 8 display time courses of these shares in the benchmark scenario and the simulated Scenarios 1, 2, and 3, respectively. The time courses are adjusted to account for the following biomass flows between sectors: biomass used in incineration includes roundwood, and byproducts from the pulp and wood industries; biomass used in the pulp industry includes roundwood and byproducts from wood industries, but excludes byproducts from pulp industries used for incineration; while biomass used in the wood industry includes roundwood, but excludes byproducts from the wood products industry used by the incineration and pulp industries.

As can be seen in Figure 5 the shares of biomass used by the three sectors are fairly constant over the model horizon[7], and the pulp, wood, and incineration sectors use around 70 %, 24 %, and 6 % of the total volume of biomass harvested, respectively.

[7]Even though the model data and parameters are not factual they have been chosen carefully to avoid excessive divergence from reality. Data from Nilsson (2006) for Sweden in the year 2004 provide aggregated biomass use shares of 64, 28, and 8 % for the pulp, wood products, and district heating and fuel pellet sectors, respectively. These figures are fairly close to those seen in the benchmark scenario, in which the pulp, wood, and incineration sectors use around 70, 24, and 6 %, of the total volume of biomass harvested, respectively.

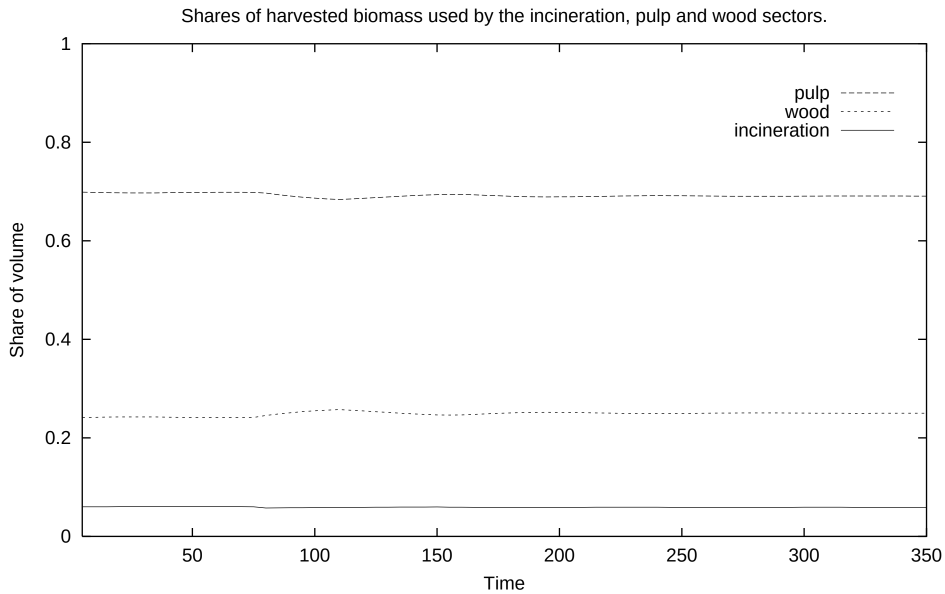

Figure 5: Aggregated proportions of harvested biomass used by the pulp, wood, and incineration sectors in the benchmark scenario, which are fairly constant over the model horizon (ca. 70, 24 and 6 %, respectively).

Figure 6 shows how the simulated biomass use changes in response to policy measures introduced in Scenario 1. When forest owners delay harvest in response to harvest taxes and growth subsidies, roundwood in the older compartments of the forest becomes relatively more abundant and less expensive than in the benchmark scenario. Users of harvested biomass from these older compartments, mainly the wood products industry, therefore increase their share of the total biomass harvested. In contrast, biomass in compartments of the forest with younger forest becomes relatively scarce and the relative price of roundwood from those compartments increases. This increase in price mostly affects the incineration and pulp industries' use of timber. The incineration industry, however, relies on residuals from the wood and pulp industries and is thus not strongly affected by the introduced policy in terms of resource allocation. The pulp industry, on the other hand, relies heavily on timber biomass and is only partly compensated by the increased flow of residuals from the wood industry, hence its use of biomass declines. Considering the timing of changes in the use of biomass, the overall trends are that the wood industry increases its use of biomass at the expense of the pulp industry's use. Note that the respective maximum and minimum shares of biomass used by the wood and pulp industries coincide with the peak average harvest age shown in Figure (3), probably due to the assumption that the wood industry relies more than the pulp industry on old biomass.

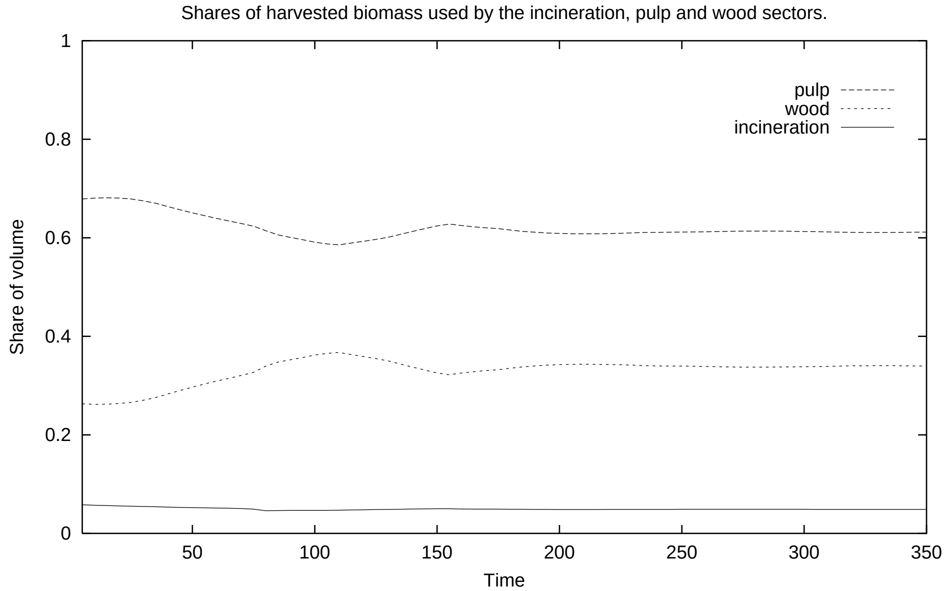

Figure 6: Aggregated shares of the total harvested biomass used by the wood, pulp and incineration sectors in Scenario 1. Biomass in the older compartments of the forest becomes relatively more abundant and less expensive in response to taxes and subsidies, than in the benchmark scenario. Users of harvested biomass from these older compartments, mainly the wood products industry, therefore increase their share of total biomass harvested. Younger compartments of the forest, on the other hand, become relatively scarcely populated with biomass and the relative price of biomass from those age classes increases, affecting primarily the incineration and pulp industries. However, the incineration industry is sheltered from this effect due to its heavy reliance on residuals from the wood and pulp industries in terms of resource allocation. The pulp industry, on the other hand, is only partly compensated by the increased flow of residuals from the wood industry and, accordingly, its use of biomass declines.

In Scenario 2, the trends in shares of biomass use, shown in Figure 7, are similar to those of Scenario 1 (illustrated in Figure 6) for reasons presented in the previous paragraph. However, the effects of the introduced policy measures on the pulp and wood industries are more pronounced, while the differentiated tax rebate applied in Scenario 2 results in even older forest than in Scenario 1. The share of biomass used by the incineration industry does not notably change, further indicating that the change in biomass use is mostly due to a transfer from the pulp industry to the wood industry.

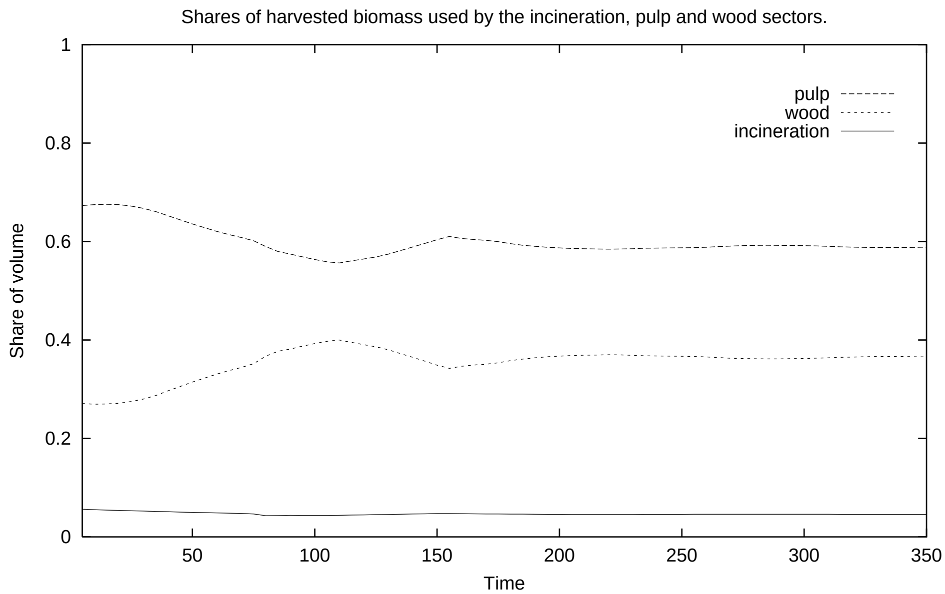

Figure 7: Aggregated shares of total harvested biomass used by the wood, pulp and incineration sectors in Scenario 2. When tax rebates targeting CO_2 storage in products are present, biomass in the older compartments of the forest becomes even more relatively abundant and cheaper than in Scenario 1. This is primarily beneficial for the wood products industry and detrimental for the pulp industry, narrowing the gap in biomass use between these two industries. As in Scenario 1, the incineration industry, is not strongly affected by the policy changes due to its reliance on residuals.

When the tax rebate on harvest is augmented with the inclusion of stumpage tax rebate in Scenario 3, shares of biomass use closely mimics those of Scenario 1 in terms of amplitude of the share values, as can be seen in Figure (8).

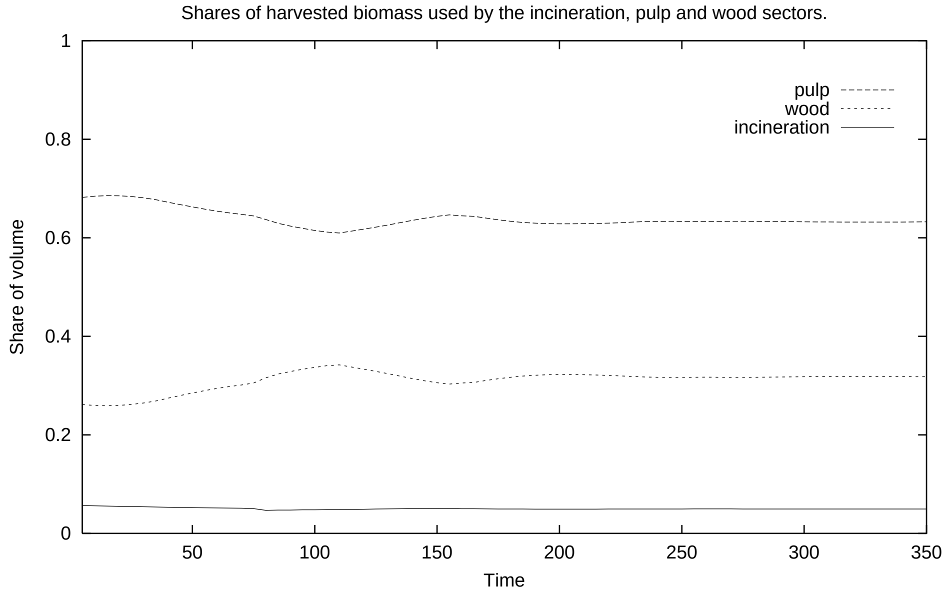

Figure 8: Aggregated shares of biomass used by the wood, pulp and incineration sectors in Scenario 3. When harvest tax rebates are augmented with a stumpage tax rebate, the amplitude of changes in the shares of biomass use closely mimic those in Scenario 1.

6.4. Income transfers

Figures 9, 10, and 11 present time courses of proportions of the total income of forest owners of harvest income, carbon growth subsidy and harvest tax, respectively. Total income is defined as harvest income plus carbon growth subsidy income minus carbon harvest tax. These shares do not sum to one since the tax has a effect on total income. The figures show the outcomes from the three scenarios apart from the benchmark scenario, in which all income stems from harvest sales for which no data are presented. All three figures show that subsidies and taxes are large, indicating that large money transfers among forest owners may occur should any of the prescribed policy scenarios be implemented.

Figure 9, shows that forest owners as a group would receive subsidies exceeding taxes paid for the first 75 years, in accordance with the initial growth of the forest illustrated in Figures 2 and 3. Between years 75 and 125 the forest owners pay more tax for harvest than they receive for subsidies accruing from growth in accordance with the rebound effect seen in Figure 3. During subsequent periods the tax payments and subsidies received are more or less equal, hence income stems solely from biomass sales. In steady state, both subsidies for growth and taxes on harvest amount to 74 % of the forest owners' income, hence they have no net effect on the owners' total income.

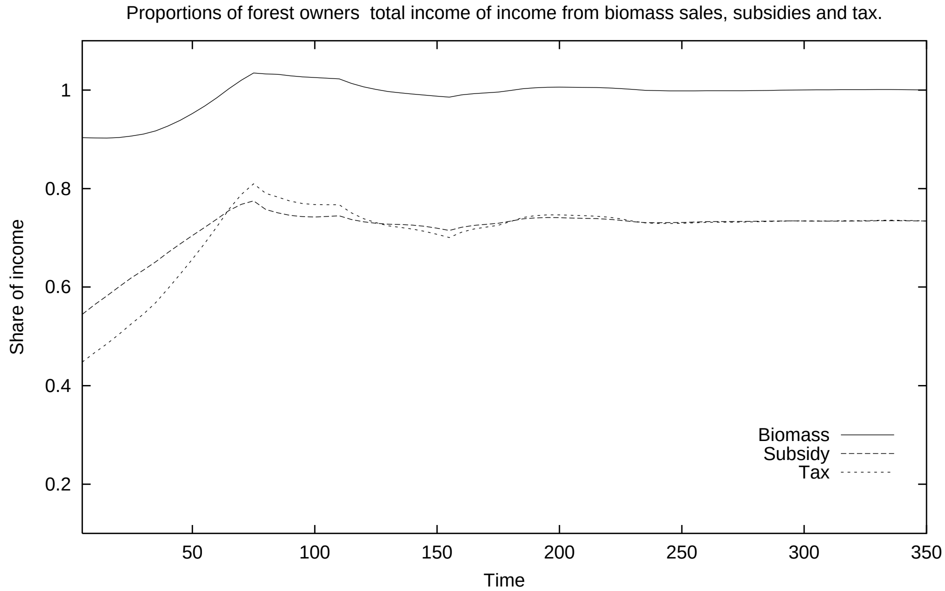

Figure 9: Changes in values of the CO_2 subsidy, CO_2 tax, and biomass sales as proportions of forest owners' income as a result of implementing policy measures in Scenario 1. As a group, forest owners receive subsidies exceeding the taxes paid for the first 75 years. Between years 75 and 125 the forest owners pay more tax for harvest than they receive from growth subsidies. During subsequent periods the tax payments and subsidies received are more or less equal, thus income stems solely from biomass sales. The shares of taxes paid on CO_2 contents of harvested biomass, and subsidies received for biomass growth are large, indicating that money transfers among forest owners might be considerable.

Figure 10 shows how subsidies exceed taxes when tax rebates are present in Scenario 2. Income from subsidies even exceed income from biomass after year 50. The tax rebate inserts a "wedge" between the amount of subsidies received and the amount of harvest taxes paid, which translates into a larger overall tax burden on the economy than in Scenario 1, in which no harvest tax rebate is allowed. The tax levied on consumption in order to pay for this "wedge" hits both capital-labor owners and forest owners to the same degree.

However, the "wedge" of course accrues to the forest owners, hence they will be better off, as shown in figures illustrating consumption levels below. In steady state, the percentages of forest owners' total income of income from biomass sold, subsidies and taxes are 77, 82 and 61 %, respectively, in Scenario 2.

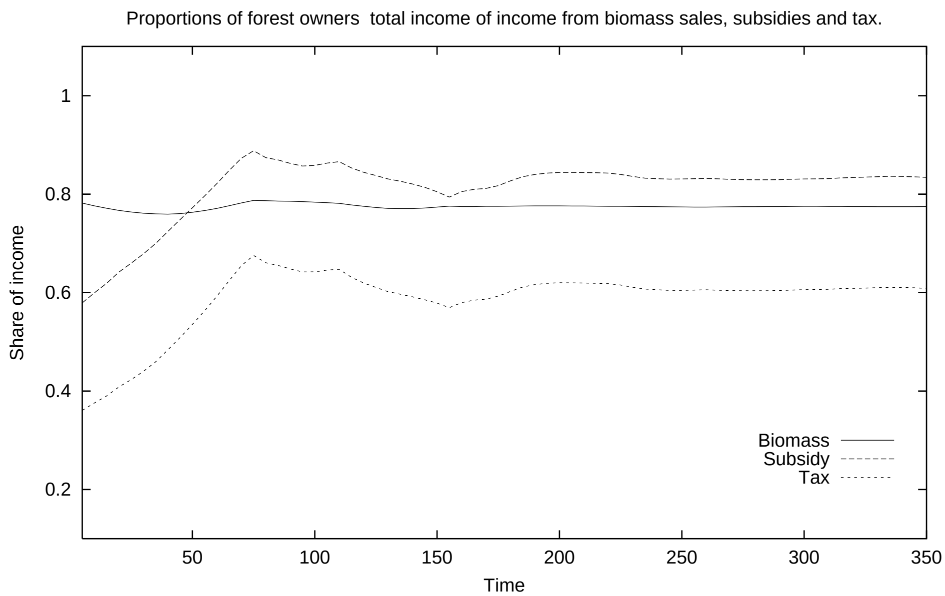

Figure 10: Changes in values of the CO_2 subsidy, CO_2 tax, and biomass sales as proportions of forest owners' income as a result of implementing policy measures in Scenario 2. Income subsidies exceed taxes and even exceed income from biomass sales after year 50. The tax rebate inserts a "wedge" between the amount of subsidies received and the amount of taxes paid for harvests, which translates into a larger overall tax burden on the economy than in Scenario 1.

When a further rebate on tax on harvest is introduced, as stipulated by Scenario 3, the "wedge" between subsidies received for forest growth and harvest taxes paid is widened compared to the situation in Scenario 2. This case is illustrated in Figure 11. As a result of the widening of the "wedge", the share of biomass income to total income shrinks compared to the share in Scenario 2. This increased discrepancy between subsidies paid for forest growth by the authority, $GOVT$, and the tax received from forest owners when harvesting, means that the tax levied on consumption has to be increased even further than in Scenario 2. Thus, the overall transfer of welfare from capital-labor owners to forest owners is even larger. As percentages of the forest owners' income in steady state, biomass, subsidies, and taxes paid amount to 55, 62, and 19 %, respectively, in Scenario 2.

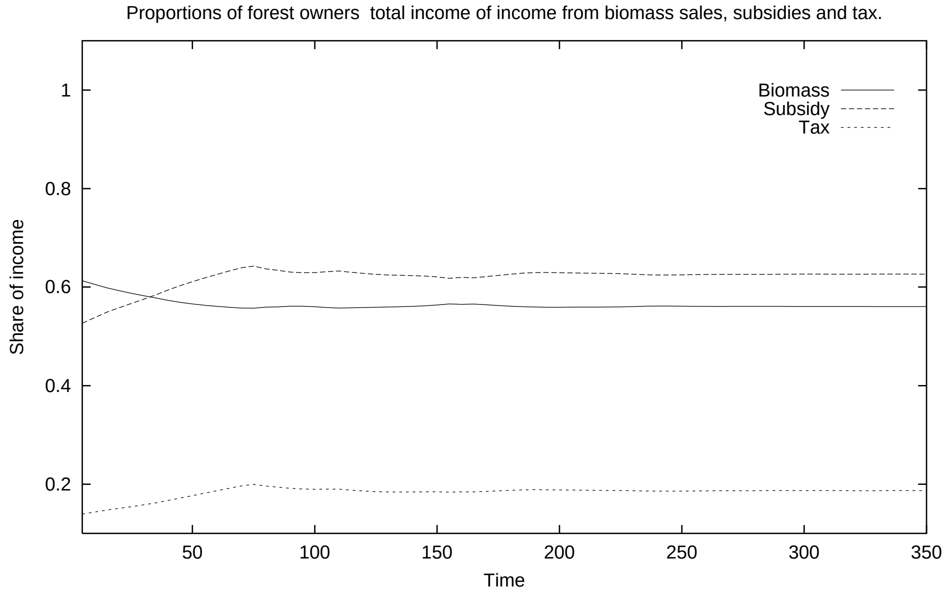

Figure 11: Changes in values of the CO_2 subsidy, CO_2 tax, and biomass sales as percentages of forest owners' income as a result of implementing policy measures in Scenario 3. The "wedge" between subsidies received for forest growth and harvest taxes paid is widened compared to the situation in Scenario 2. As a result of the widening of the "wedge", the share of biomass income to total income shrinks compared to Scenario 2. This increased discrepancy between subsidies paid for forest growth by the authority, $GOVT$, and the tax received from forest owners when harvesting, means that the tax levied on consumption has to be increased even further than in Scenario 2. Thus, the overall tax burden on the economy is even larger.

Figures 12 and 13, presents the simulated consumption paths of the forest and capital-labor owners relative to their respective benchmark consumption paths. As an indication of the relative consumption power between the two consumer groups, the consumption of the group capital-labor owners in the benchmark is 6.6 times greater than the forest owners' consumption.

Comparing the two figures, two aspects are noteworthy. First, for each scenario an increase of consumption for forest owners is matched by a decrease in consumption for capital-labor owners across the whole model horizon. Thus, there is a transfer, compared to the benchmark scenario, of consumption from capital-labor owners to forest owners. This transfer is largest for Scenario 3 in which the policy allows for highest tax reductions for the forest owners, and lowest for Scenario 1 where no tax reductions are present.

Second, the level of consumption of both capital-labor owners and forest owners increases during the first 75 years in all three scenarios. This indicates that the dead weight losses associated with taxes and subsidies are larger in this time-frame while the age-structured forest-harvest system accommodates the policy induced disturbance.

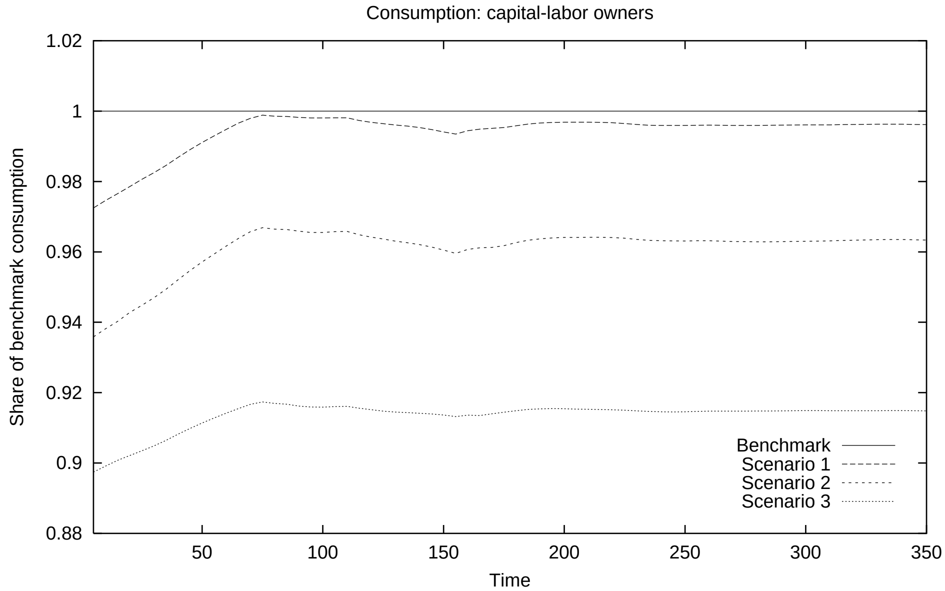

Figure 12: Consumption by capital-labor owners, as a proportion of benchmark consumption. Their consumption for is lower, across the whole model horizon, in Scenarios 1,2 and 3 than in the benchmark scenario, and these are matched by consumption increases for the forest owners. Hence, there is a transfer, compared to the benchmark scenario, of consumption from capital-labor owners to forest owners. This transfer is largest for Scenario 3, in which the policy measures provide the highest tax reductions for the forest owners, and lowest for Scenario 1, where no tax reductions are present.

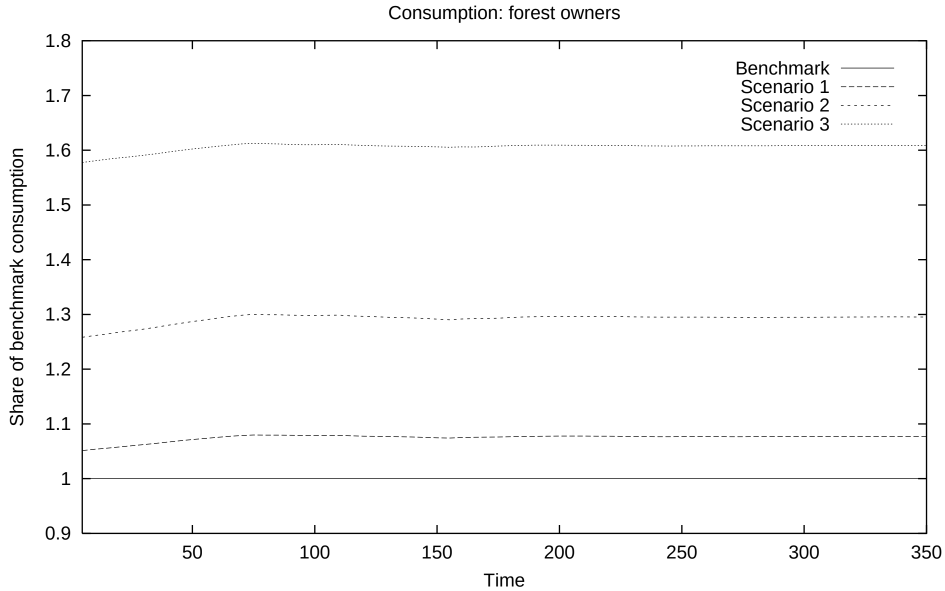

Figure 13: Consumption by forest owners, as a proportion of benchmark consumption. Their consumption is lower, across the whole model horizon, in Scenarios 1,2 and 3 than in the benchmark scenario, matched by a reductions in consumption for capital-labor owners. Hence, there is a transfer, compared to the benchmark scenario, of consumption from capital-labor owners to forest owners. This transfer is largest for Scenario 3, in which the policy measures provide the highest tax reductions for the forest owners, and lowest for Scenario 1, where no tax reductions are present.

The results presented in this section are summarized in Table 1, which provides a snapshot of the steady state situation. The table shows that average forest age, average harvest age, and the amount of carbon sequestrated all increase in the scenarios, relative to the benchmark scenario. The increments are largest for Scenario 2. The table also shows that there would be large money transfers among forest owners since the values of the harvest tax and growth subsidies as proportions of total income tend to be considerable and the forest biomass use shifts from the pulp industry to the wood industry in all simulated scenarios. The incineration industry, on the other hand, seems not to be affected in terms of biomass supply. Further, the introduced policies cause welfare transfers to forest owners, which are enhanced when tax rebates are established. Harvest tax rebates targeting biomass used in long-lived products tend to increase total CO_2 storage, but an addition of a more general rebate might decrease total CO_2 storage compared to no rebate at all. The introduction of tax rebates increases the social cost of CO_2 storage-increasing policies, and the most expensive is the general tax rebate, which also might result in lower CO_2 storage.

Summary of results				
	Benchmark	Scenario 1	Scenario 2	Scenario 3
Average forest age*	0	6.23	7.92	4.72
Average harvest age*	0	5.94	7.82	4.26
Carbon capture*	0	12.5	16	9.5
Biomass use Wood	26	34	36	32
Biomass use Pulp	69	61	59	63
Biomass use Incineration	5	5	5	5
Share of income from biomass	100	100	77	55
Share of income from subsidies	0	74	82	62
Share of income tax	0	74	61	19
Consumption capital-labor owners*	0	-0.5	-3.7	-8.5
Consumption forest owners*	0	8	30	61

Table 1: Summary of steady state results. All figures are percentages. Asterisks indicate percentage changes relative to the benchmark scenario.

7. Conclusion

This paper proposes a dynamic CGE model describing renewable biomass stocks in forests, and effects of their growth and harvests on other economic activities. Particular attention is paid to age-specific properties of the biomass stock. Harvests of, and the demand for, the renewable biomass stocks are determined endogenously in an inter-temporally consistent way.

To illustrate the application of the model framework, simulations are presented of effects of selected policy scenarios designed to increase carbon sequestration in forest biomass. The scenarios are based on taxing harvest and subsidizing forest growth, which induces prolongation of forest growth before harvest, and hence in the amount of CO_2 stored in the productive forest area. In addition, effects of two rebate programs of the introduced harvest tax are analyzed.

The results suggest that the cost to society of the investigated policy scenarios, would be larger in the beginning of the planning horizon, but there might be a considerable delay before carbon storage in the forest increased significantly. The analysis of the examined policy scenarios also indicates that large money transfers among the forest owners are likely to occur as the values of the harvest tax and growth subsidies relative to total income tend to be considerable.

The analysis indicates that biomass use is likely to shift from the pulp industry towards the wood industry, due to differences in the age-specific types of biomass used by the industries. This is intuitively plausible, since the sequestration policy makes the forest older.

The results also suggest that introduced policies would induce wealth transfer to forest owners from other economic agents. These transfers are likely to be further enhanced if tax rebates are established. Tax rebates targeting CO_2 storage in products (Scenario 2), tend to increase total CO_2 storage, while a tax rebate directed at CO_2 storage in stumpage left in the forest (Scenario 3) might decrease total CO_2 storage. The introduction of tax rebates increases social costs of these CO_2 storage-increasing policies, and the most expensive, Scenario 3, might also result in lower CO_2 storage.

Artificial data regarding dynamic changes in the forest age structure (state, growth, and harvest), and the age-structure of biomass used by the considered industrial sectors were applied in the modeling. These data are rarely available, at this fine level of detail, in national forest inventories, and national accounting. Therefore, a method for extrapolating a consistent age-structured data set describing the state, growth, and harvest of the forest, together with econometric examination of the age-structured biomass used in relevant sectors, is suggested for future studies to provide more robust evidence when using this framework for empirical studies. Another shortcoming of the model presented is the implicit assumption of constant stocks of capital and labor. A more realistic framework would also account for investment and depreciation of capital, and changes in the labor force. Furthermore, for completion, the ecological part of the model should account for possible changes in soil carbon stocks.

Appendix A. Model equations

Since the age-structure of the forest and the uses of forest products have major effects on the outcomes of the model, more detailed discussion of how these variables are described is presented in this appendix. Timber from trees of different age classes is assumed to find different uses in diverse sectors of the economy, and some assumptions have to be made regarding the allocation of different biomass resources in the economy. The more specific considerations are as follows.

1. Harvested roundwood is considered sector-specific in the sense that different sectors use biomass from trees of specific age classes. More specifically, the incineration, pulp and wood product sectors are considered to use, respectively: primarily roundwood from some of the youngest harvested age-classes, a_i; primarily roundwood from intermediate age-classes, a_p; and solely the oldest biomass, a_w.
2. The sets a_i, a_p, and a_w are assumed to overlap, such that a_i is totally within a_p ($a_i \subset a_p$), a_i has no element in common with a_w ($a_i \cap a_w = \emptyset$), and a_p has some elements in common with a_w. The two sets of overlapping age-classes are termed $a_{ip} = a_i \cap a_p$, for the overlap between roundwood used in incineration and pulp industries, and $a_{pw} = a_p \cap a_w$, for the overlap between timber used by the pulp and wood industries. The remaining non-overlapping age classes are termed $a_{pp} = a_p - a_{pw}$ and $a_{ww} = a_w - a_{pw}$ for age classes of roundwood used exclusively by

the pulp industry and wood industry, respectively.

3. The overlapping sets, a_{ip} and a_{pw}, of roundwood, finding use in distinct sectors, are assumed to be subject to an inter-sector specific elasticity of transformation.
4. Within sectors using roundwood, substitution among the different age-classes of roundwood used is assumed to occur.
5. Residues from the wood industry are used as biomass input in both the incineration and pulp industries, while residues from the pulp industries are used in incineration.

In order to accommodate these considerations a set of production activities are set up. These activities represent a subdivision of the activity described for the zero-profit condition 5.1 in section 5.2, and are stated as:

Roundwood inertia These activities are aggregated in order to model the assumed partly restricted choices faced by forest owners as a group, regarding where to ship their harvested timber. The restrictions are modeled as elasticities of transformation between the choices, in the revenue functions associated with these activities. These activities also enable effects of tax rebates targeting biomass use to be investigated, as in Scenarios 2 and 3. The levels of these activities are denoted by the following variables:

$\mathbf{b}_{t,a_{ip}}$ representing the volume of roundwood used by both incineration and pulp industries,

$\mathbf{b}_{t,a_{pw}}$ denoting the volume of roundwood used by both pulp and wood industries,

$\mathbf{b}_{t,a_{pp}}$ specifying the volume of roundwood used solely by the pulp industry, and,

$\mathbf{b}_{t,a_{ww}}$ constituting the volume of roundwood used by solely by the wood industry.

Biomass production These activities model the process of transforming harvested timber to an aggregated multi-biomass output using capital and labor. The resulting output is the biomass actually used in the end biomass products. The value-added multi-output character of the activities enables the modeling of use of byproducts resulting from main production. The levels of these activities are denoted by the following variables:

b_t^i representing the activity level of producing biomass from timber for the incineration industry,

b_t^p denoting the activity level of producing biomass from timber for the pulp and incineration industries,

b_t^w constituting the activity level of producing biomass from timber for the wood, pulp, and incineration industries.

Top level biomass Activity 5.1 is redefined by using an aggregated biomass input instead of roundwood. The variable describing the activity levels,

$\mathbf{x}_t^f$, is the same as before.

In addition to the variables associated with activity levels delineated above, the following additional variables for prices are needed:

$\mathbf{p}^b_{t,a_{ip}}$ The prices for roundwood used by both incineration and pulp industries that forest owners receive,

$\mathbf{p}^b_{t,a_{pp}}$ prices for roundwood used solely by pulp industries that forest owners receive,

$\mathbf{p}^b_{t,a_{pw}}$ prices for roundwood used by both pulp and wood industries that forest owners receive,

$\mathbf{p}^b_{t,a_{ww}}$ prices for roundwood used solely by wood industries that forest owners receive,

$\mathbf{p}^{bi}_{t,a_i}$ prices for roundwood paid by the incineration industry,

$\mathbf{p}^{bp}_{t,a_p}$ prices for roundwood paid by pulp industry,

$\mathbf{p}^{bw}_{t,a_w}$ prices for roundwood paid by wood industry,

p^i_t price index for value-added biomass used in the incineration industry,

p^p_t price index for value-added biomass used in the pulp industry,

p^w_t price index for value-added biomass used in the wood industry.

Appendix A.1. New zero-profit conditions

For overlapping age classes we have the inter-sectoral zero-profit relations for the *roundwood inertia* activities:

$$\mathbf{p}^b_{t,a_{ip}} \geq \mathbf{r}^b_{t,a_{ip}}(\mathbf{p}^{bi}_{t,a_{ip}}, \mathbf{p}^{bp}_{t,a_{ip}} + \kappa_{a_p} p^{CO_2}_t) \qquad \perp \qquad \mathbf{b}_{t,a_{ip}} \geq \mathbf{0}$$

$$\mathbf{p}^b_{t,a_{pw}} \geq \mathbf{r}^b_{t,a_{pw}}(\mathbf{p}^{bp}_{t,a_{pw}} + \kappa_{a_p} p^{CO_2}_t, \mathbf{p}^{bw}_{t,a_{pw}} + \kappa_{a_w} p^{CO_2}_t) \qquad \perp \qquad \mathbf{b}_{t,a_{pw}} \geq \mathbf{0}$$

where $\mathbf{r}^b_{t,a_{ip}}(\cdot)$ and $\mathbf{r}^b_{t,a_{pw}}(\cdot)$ are the revenue functions taking care of consideration 3.

For non overlapping age classes we have the inter-sectoral zero-profit relations for the remaining *roundwood inertia* activities:

$$\mathbf{p}^b_{t,a_{pp}} = \mathbf{p}^{bp}_{t,a_{pp}} + \kappa_{a_p} p^{CO_2}_t \qquad \perp \qquad \mathbf{b}_{t,a_{pp}} \geq \mathbf{0}$$

$$\mathbf{p}^b_{t,a_{ww}} = \mathbf{p}^{bw}_{t,a_{ww}} + \kappa_{a_p} p^{CO_2}_t \qquad \perp \qquad \mathbf{b}_{t,a_{ww}} \geq \mathbf{0}$$

Note that the tax rebate parameters enter these zero-profit conditions and enable investigation of Scenarios 2 and 3.

Activities labeled *biomass production* use capital, labor, and roundwood as input. The zero-profit conditions for these activities are:

$$c^i_t(p^K_t, p^L_t, \mathbf{p}^{bi}_{t,a_i}) \geq p^i_t \qquad \perp \qquad b^i_t \geq 0$$

$$c^p_t(p^K_t, p^L_t, \mathbf{p}^{bp}_{t,a_p}) \geq r^p_t(p^i_t, p^p_t) \qquad \perp \qquad b^p_t \geq 0$$

$$c_t^i(p_t^K, p_t^L, \mathbf{p}_{t,a_w}^{bw}) \geq r_t^w(p_t^i, p_t^p, p_t^w) \qquad \perp \qquad b_t^w \geq 0$$

where $c_t^i(\cdot)$, $c_t^p(\cdot)$, and $c_t^w(\cdot)$ are the cost functions associated with production of value-added biomass in the incineration-, pulp-, and wood industries. These cost functions address consideration 4. The revenue functions $r_t^p(\cdot)$ and $r_t^w(\cdot)$ handle consideration 5.

Top level biomass goods are now produced from value added biomass, capital and labor:

$$c_t^{x_i}(p_t^K, p_t^L, p_t^i) \geq p_t^{x_i} \qquad \perp \qquad x_t^i \geq 0$$

$$c_t^{x_p}(p_t^K, p_t^L, p_t^p) \geq p_t^{x_p} \qquad \perp \qquad x_t^p \geq 0$$

$$c_t^{x_w}(p_t^K, p_t^L, p_t^w) \geq p_t^{x_w} \qquad \perp \qquad x_t^w \geq 0$$

Appendix A.2. New market clearance conditions

With the separation of harvested roundwood into overlapping and non-overlapping sets, the market clearance condition for the volume of biomass harvested, 5.24, now reads:

$$\mathbf{Vh}_t \geq \begin{bmatrix} \mathbf{b}'_{t,a_{ip}} & \mathbf{b}'_{t,a_{pp}} & \mathbf{b}'_{t,a_{pw}} & \mathbf{b}'_{t,a_{ww}} \end{bmatrix}' \qquad \perp \qquad \mathbf{p}_t^b \geq 0$$

If prices for factors and goods are determined in competitive markets, balance of supply and demand is assured. Using Shepard's lemma, we can derive the market clearance conditions of the remaining new activities:

$$\mathbf{b}'_{t,a_{ip}} \nabla_{\mathbf{p}^{bi}} \mathbf{r}^b_{t,a_{ip}}(\cdot) \geq b^i_t \nabla_{\mathbf{p}^{bi}} c^i_t(\cdot) \qquad \perp \qquad \mathbf{p}^{bi}_{t,a_{ip}} \geq \mathbf{0}$$

$$\begin{bmatrix} \mathbf{b}'_{t,a_{ip}} \nabla_{\mathbf{p}^{bp}} \mathbf{r}^b_{t,a_{ip}}(\cdot) \\ \mathbf{b}_{t,a_{pp}} \\ \mathbf{b}'_{t,a_{pw}} \nabla_{\mathbf{p}^{bp}} \mathbf{r}^b_{t,a_{pw}}(\cdot) \end{bmatrix} \geq b^p_t \nabla_{\mathbf{p}^{bp}} c^p_t(\cdot) \qquad \perp \qquad \mathbf{p}^{bp}_{t,a_p} \geq \mathbf{0}$$

$$\begin{bmatrix} \mathbf{b}'_{t,a_{pw}} \nabla_{\mathbf{p}^{bw}} \mathbf{r}^b_{t,a_{pw}}(\cdot) \\ \mathbf{b}_{t,a_{ww}} \end{bmatrix} \geq b^w_t \nabla_{\mathbf{p}^{bw}} c^w_t(\cdot) \qquad \perp \qquad \mathbf{p}^{bw}_{t,a_w} \geq \mathbf{0}$$

$$b^i_t + [b^p_t \ b^w_t] \frac{\partial}{\partial p^i_t} \begin{bmatrix} r^p_t(\cdot) \\ r^w_t(\cdot) \end{bmatrix} \geq x^i_t \frac{\partial c^{x_i}_t(\cdot)}{\partial p^i_t} \qquad \perp \qquad p^i_t \geq 0$$

$$[b^p_t \ b^w_t] \frac{\partial}{\partial p^p_t} \begin{bmatrix} r^p_t(\cdot) \\ r^w_t(\cdot) \end{bmatrix} \geq x^p_t \frac{\partial c^{x_p}_t(\cdot)}{\partial p^p_t} \qquad \perp \qquad p^p_t \geq 0$$

$$b^w_t \frac{\partial r^w_t(\cdot)}{\partial p^w_t} \geq x^w_t \frac{\partial c^w_t(\cdot)}{\partial p^w_t} \qquad \perp \qquad p^w_t \geq 0$$

Given that the new activities use capital and labor, the market clearance conditions for those markets need to be rectified:

$$K_t \geq \left[b_t^i \; b_t^p \; b_t^w \; x_t^i \; x_t^p \; x_t^w \; x_t^o \right] \frac{\partial}{\partial p_t^K} \begin{bmatrix} c_t^i(\cdot) \\ c_t^p(\cdot) \\ c_t^w(\cdot) \\ c_t^{x_i}(\cdot) \\ c_t^{x_p}(\cdot) \\ c_t^{x_w}(\cdot) \\ c_t^{x_o}(\cdot) \end{bmatrix} \qquad \perp \qquad p_t^K \geq 0$$

$$L_t \geq \left[b_t^i \; b_t^p \; b_t^w \; x_t^i \; x_t^p \; x_t^w \; x_t^o \right] \frac{\partial}{\partial p_t^L} \begin{bmatrix} c_t^i(\cdot) \\ c_t^p(\cdot) \\ c_t^w(\cdot) \\ c_t^{x_i}(\cdot) \\ c_t^{x_p}(\cdot) \\ c_t^{x_w}(\cdot) \\ c_t^{x_o}(\cdot) \end{bmatrix} \qquad \perp \qquad p_t^L \geq 0$$

Appendix A.3. Functional forms

The functional forms and parameters of the model are given below for completeness.

$$r_{t,a_{ip}}^b(p_{t,a_{ip}}^{bi}, p_{t,a_{ip}}^{\kappa bp}) = \left(\theta_{a_{ip}} (p_{t,a_{ip}}^{bi})^{1+\alpha_{a_{ip}}} + (1-\theta_{a_{ip}})(p_{t,a_{ip}}^{\kappa bp})^{1+\alpha_{a_{ip}}} \right)^{\frac{1}{1+\alpha_{a_{ip}}}}$$

$$p^{\kappa bp}_{t,a_{ip}} = p^{bp}_{t,a_{ip}} + \kappa_{a_p} p^{CO_2}_t$$

$$r^b_{t,a_{pw}}(p^{\kappa bp}_{t,a_{ip}}, p^{\kappa bw}_{t,a_{pw}}) = \left(\theta_{a_{pw}}(p^{\kappa bp}_{t,a_{ip}})^{1+\alpha_{a_{pw}}} + (1-\theta_{a_{pw}})(p^{\kappa bw}_{t,a_{pw}})^{1+\alpha_{a_{pw}}}\right)^{\frac{1}{1+\alpha_{a_{pw}}}}$$

$$p^{\kappa bw}_{t,a_{pw}} = p^{bw}_{t,a_{pw}} + \kappa_{a_w} p^{CO_2}_t$$

$$c^i_t(p^K_t, p^L_t, \mathbf{p}^{bi}_{t,a_i}) = \left[\begin{array}{l} \theta^i_{va}\left(\theta^i_K\left(p^K_t\right)^{1-\sigma^i_{va}} + (1-\theta^i_K)\left(p^L_t\right)^{1-\sigma^i_{va}}\right)^{\frac{1-\sigma^i}{1-\sigma^i_{va}}} \\ + (1-\theta^i_{va})\left(\sum_{a_i}\theta^i_{a_i}\left(p^{bi}_{t,a_i}\right)^{1-\sigma^i_b}\right)^{\frac{1-\sigma^i}{1-\sigma^i_b}} \end{array}\right]^{\frac{1}{1-\sigma^i}}$$

$$c^p_t(p^K_t, p^L_t, \mathbf{p}^{bp}_{t,a_p}) = \left[\begin{array}{l} \theta^p_{va}\left(\theta^p_K\left(p^K_t\right)^{1-\sigma^p_{va}} + (1-\theta^p_K)\left(p^L_t\right)^{1-\sigma^p_{va}}\right)^{\frac{1-\sigma^p}{1-\sigma^p_{va}}} \\ + (1-\theta^p_{va})\left(\sum_{a_p}\theta^p_{a_p}\left(p^{bp}_{t,a_p}\right)^{1-\sigma^p_b}\right)^{\frac{1-\sigma^p}{1-\sigma^p_b}} \end{array}\right]^{\frac{1}{1-\sigma^p}}$$

$$c^w_t(p^K_t, p^L_t, \mathbf{p}^{bw}_{t,a_w}) = \left[\begin{array}{l} \theta^w_{va}\left(\theta^w_K\left(p^K_t\right)^{1-\sigma^w_{va}} + (1-\theta^w_K)\left(p^L_t\right)^{1-\sigma^w_{va}}\right)^{\frac{1-\sigma^w}{1-\sigma^w_{va}}} \\ + (1-\theta^w_{va})\left(\sum_{a_w}\theta^w_{a_w}\left(p^{bw}_{t,a_w}\right)^{1-\sigma^w_b}\right)^{\frac{1-\sigma^w}{1-\sigma^w_b}} \end{array}\right]^{\frac{1}{1-\sigma^w}}$$

$$r^p_t(p^i_t, p^p_t) = \left((1-\theta^p)(p^i_t)^{1+\alpha_p} + \theta^p(p^p_t)^{1+\alpha_p}\right)^{\frac{1}{1+\alpha_p}}$$

$$r^w_t(p^i_t, p^p_t, p^w_t) = \left(\theta^i(p^i_t)^{1+\alpha_w} + (1-\theta^i-\theta^w)(p^p_t)^{1+\alpha_w} + \theta^w(p^w_t)^{1+\alpha_w}\right)^{\frac{1}{1+\alpha_w}}$$

$$c^{x_i}_t(p^K_t, p^L_t, p^i_t) = \left[\begin{array}{l} \theta^{x_i}_{va}\left(\theta^{x_i}_K\left(p^K_t\right)^{1-\sigma^{x_i}_{va}} + (1-\theta^{x_i}_K)\left(p^L_t\right)^{1-\sigma^{x_i}_{va}}\right)^{\frac{1-\sigma^{x_i}}{1-\sigma^{x_i}_{va}}} \\ + (1-\theta^{x_i}_{va})\left(p^i_t\right)^{1-\sigma^{x_i}} \end{array}\right]^{\frac{1}{1-\sigma^{x_i}}}$$

$$c^{x_p}_t(p^K_t, p^L_t, p^p_t) = \left[\begin{array}{l} \theta^{x_p}_{va}\left(\theta^{x_p}_K\left(p^K_t\right)^{1-\sigma^{x_p}_{va}} + \left(1-\theta^{x_p}_K\right)\left(p^L_t\right)^{1-\sigma^{x_p}_{va}}\right)^{\frac{1-\sigma^{x_p}}{1-\sigma^{x_p}_{va}}} \\ + (1-\theta^{x_p}_{va})\left(p^p_t\right)^{1-\sigma^{x_p}} \end{array}\right]^{\frac{1}{1-\sigma^{x_p}}}$$

$$c_t^{x_w}(p_t^K, p_t^L, p_t^w) = \left[\begin{array}{l} \theta_{va}^{x_w} \left(\theta_K^{x_w} \left(p_t^K\right)^{1-\sigma_{va}^{x_w}} + (1-\theta_K^{x_w}) \left(p_t^L\right)^{1-\sigma_{va}^{x_w}}\right)^{\frac{1-\sigma^{x_w}}{1-\sigma_{va}^{x_w}}} \\ + (1-\theta_{va}^{x_w}) \left(p_t^w\right)^{1-\sigma^{x_w}} \end{array} \right]^{\frac{1}{1-\sigma^{x_w}}}$$

$$c_t^{x_o}(p_t^K, p_t^L) = \left(p_t^K\right)^{\theta_K^o} \left(p_t^L\right)^{1-\theta_K^o}$$

$$c_t^{x}(p_t^{x_i}, p_t^{x_p}, p_t^{x_w}, p_t^{x_o}) = (p_t^{x_i})^{\theta_{x_i}} (p_t^{x_p})^{\theta_{x_p}} (p_t^{x_w})^{\theta_{x_w}} (p_t^{x_o})^{1-\theta_{x_i}-\theta_{x_p}-\theta_{x_w}}$$

$$e^{FO}(\mathbf{p}^x) = e^{KLO}(\mathbf{p}^x) = \left(\sum_t^T \left(\frac{1}{1+\rho}\right)^{t\left(1-\frac{\eta}{\eta-1}\right)} (p_t^x)^{\frac{\eta}{\eta-1}} \right)^{\frac{\eta-1}{\eta}}$$

Parameters:

$\theta_{a_{ip}}$ revenue share parameter for roundwood targeting incineration biomass production,

$\alpha_{a_{ip}}$ elasticity of transformation between roundwood targeting incineration biomass production and roundwood targeting pulp biomass production,

$\theta_{a_{pw}}$ revenue share parameter for roundwood targeting pulp biomass production,

$\alpha_{a_{pw}}$ elasticity of transformation between roundwood targeting pulp biomass production and roundwood targeting wood biomass production,

θ_{va}^i cost share parameter for value-added nest in incineration biomass production,

θ_K^i cost share parameter for capital in incineration biomass production,

$\theta^i_{a_i}$ cost share parameters for age-structured roundwood used in incineration biomass production ($\sum_{a_i} \theta^i_{a_i} = 1$),

σ^i_{va} elasticity of substitution between capital and labor in incineration biomass production,

σ^i_b elasticity of substitution in age-structured roundwood in incineration biomass production,

σ^i elasticity of substitution between value-added nest and round wood nest, in incineration biomass production,

θ^p_{va} cost share parameter for value-added nest in pulp biomass production,

θ^p_K cost share parameter for capital in pulp biomass production,

$\theta^p_{a_i}$ cost share parameters for age structured roundwood in pulp biomass production ($\sum_{a_i} \theta^i_{a_i} = 1$),

σ^p_{va} elasticity of substitution between capital and labor in pulp biomass production,

σ^p_b elasticity of substitution in age-structured roundwood in pulp biomass production,

σ^p elasticity of substitution between value added nest and round wood nest, in pulp biomass production,

θ^w_{va} cost share parameter for value-added nest in wood biomass production,

θ^w_K cost share parameter for capital in wood biomass production,

$\theta^w_{a_i}$ cost share parameters for age structured roundwood in wood biomass production ($\sum_{a_i} \theta^i_{a_i} = 1$),

σ^w_{va} elasticity of substitution between capital and labor in wood biomass production,

σ^w_b elasticity of substitution in age-structured roundwood in wood biomass production,

σ^w elasticity of substitution between value-added nest and roundwood nest, in wood biomass production,

θ^p revenue share parameter for biomass output targeting the incineration industry,

α^p elasticity of transformation between biomass output targeting the incineration and pulp industries,

θ^i revenue share parameter for biomass output targeting the incineration industry,

θ^w revenue share parameter for biomass output targeting the wood industry,

α^w elasticity of transformation among biomass outputs targeting the incineration, pulp, and wood industries, respectively,

$\theta^{x_i}_{va}$ cost share parameter for value-added nest in the incineration industry,

$\theta_K^{x_i}$ cost share parameter for capital in the incineration industry,

$\sigma_{va}^{x_i}$ elasticity of substitution between capital and labor in the incineration industry,

σ^{x_i} elasticity of substitution between value-added nest and biomass in the incineration industry,

$\theta_{va}^{x_p}$ cost share parameter for value-added nest in the pulp industry,

$\theta_K^{x_p}$ cost share parameter for capital in the pulp industry,

$\sigma_{va}^{x_p}$ elasticity of substitution between capital and labor in the pulp industry,

σ^{x_p} elasticity of substitution between value-added nest and biomass in the pulp industry,

$\theta_{va}^{x_w}$ cost share parameter for value-added nest in the wood industry,

$\theta_K^{x_w}$ cost share parameter for capital in the wood industry,

$\sigma_{va}^{x_w}$ elasticity of substitution between capital and labor in the wood industry,

σ^{x_w} elasticity of substitution between value-added nest and biomass in the wood industry,

θ_K^{o} cost share parameter for capital in rest of industry,

θ_{x_i} cost share parameter of incineration in intra-period consumption,

θ_{x_p} cost share parameter of pulp in intra-period consumption,

θ_{x_w} cost share parameter of wood in intra-period consumption.

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