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CERE Working Paper, 2012:17

Regulation and Unintended Consequences

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August 2012

Introduction

Production of desirable outputs such as Kwh of electricity are often accompanied by the production of undesirable or 'bad' outputs such as SO_2. These undesirable outputs are frequently regulated in the sense that their production is not allowed to exceed certain amounts.

In this paper we analyze what we call the unintended consequences of regulation of bads where that regulation limits the quantity of bads produced. We consider the simple case in which there is one good and one bad output. Under constant returns to scale we provide a theorem that characterizes the situation in which quantity regulation of the bad output restricts the production of the intended good output. Our theorem is in the spirit of Shephard's proof of the Law of Diminishing Returns, see Shephard (1970:a).[1]

1 The Main Findings

We begin with the axiomatic framework of technology in which good and bad outputs are jointly produced.[2] In terms of notation, let $y \geqq 0$ denote the single desirable output and $b \geqq 0$ the only undesirable output. Inputs are denoted by $x = (x_1, \ldots, x_N) \in \Re^N_+$.

The technology is modeled by its output correspondence

$$P(x) = \{(y, b) : x \text{ can produce } (y, b)\}, x \in \Re^N_+. \tag{1}$$

Following Shephard (1970:b) we make the following assumptions on technology $P(x)$

- $P(0) = 0$, inactivity.
- $P(x)$ is bounded for all $x \in \Re^N_+$, scarcity.
- $P(x') \supseteq P(x)$ when $x' \geqq x$, strong disposability of inputs.
- $(y, b) \in P(x)$ and $0 \leqq \lambda \leqq 1 \Rightarrow (\lambda y, \lambda b) \in P(x)$, weak disposability of outputs.
- $P(x)$ is closed and nonempty.

The axioms itemized above on our technology are—with the exception of weak disposability of outputs—consistent with traditional production theory. These are discussed in detail elsewhere, for example see in Färe and Grosskopf (2004). Here we also assume that $P(x)$ is homogeneous of degree +1 (constant returns to scale).

[1] For a survey of this law, see Färe (1980).

[2] The discussion of technology with good and bad outputs closely follows Färe, Grosskopf and Pasurka (forthcoming).

One of the distinguishing features of production in the presence of good and bad outputs is based on thermodynamics; as Baumgärtner et al (2001, p. 365) state

'...the production of wanted goods gives rise to additional unwanted outputs...'

which states that bad outputs are essential byproducts of production of good outputs. Shephard and Färe (1974) model this condition by introducing what they call *Null Joint production*, i.e.,

If $(y, b) \in P(x)$, and $b = 0$ then $y = 0$.

This says that if production is null joint, in order to produce good output, some bad output byproduct will also be produced—no fire without smoke.

A simple example which satisfies our axioms is

$$P(x) = \{(y, b) : \begin{array}{ll} y + (b - 1/2y) \geqq x & \text{if } (b - 1/2y) \geqq 0 \\ 0 & \text{otherwise}\}. \end{array} \quad (2)$$

The following figure illustrates.

This output set has input $x = 1$, and the boundary of the set is the triangle 0a1. Note that this satisfies Null Jointness since if $b = 0$ then for (y, b) to be feasible, it must be true that $y = 0$.

We will eventually show that bounds on bad production limit good output production as well. To model this idea we introduce the following definition:

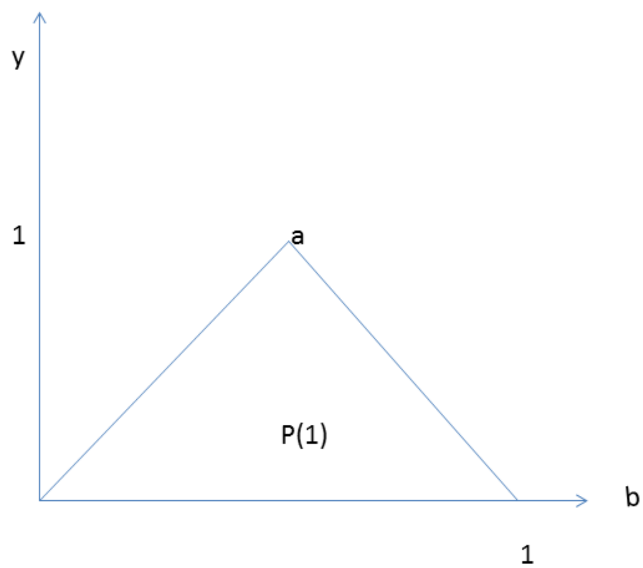
The bad output is *limitational* [3] if for all $\bar{b} \geqq 0$:

$$\sup_{x \in \Re_+^N} \{y : (y, b) \in P(x), b \leqq \bar{b}\} < +\infty. \quad (3)$$

In words, if the bad output may not exceed $\bar{b}$, the 'largest' feasible production of good output is finite with any level of input consistent with $x \in \Re_+^N$. Thus limitationality models the question we are interested in: do regulations limiting the production of bad outputs affect production of the good output? If bads are limitational, the answer is yes. The following theorem characterizes the limitationality of b.

[3] This notion is closely related to the concept of limitational inputs, see Shephard (1970:a,b).

Figure 1: An Output Set with Weakly Disposable and Null Joint Outputs



Theorem: Let $P(x)$ be homogeneous of degree $+1$ and satisfy the Shephard axioms above, then b is limitational if and only if y and b are nulljont.

Proof: (limitationality $\Rightarrow$ nulljointness) We prove its equivalent statement:

(y, b) are not nulljoint $\Rightarrow$ b is not limitational.

Assume that (y, b) are not nulljoint, then there exists a vector

$(y, b) \in P(x)$ such that $y > 0$ and $b = 0$. Note that $b = 0 \leqq \bar{b}$. Let $\lambda > 0$, then $(\lambda y, \lambda b) \in \lambda P(x) = P(\lambda x)$, for all $\lambda > 0$.

Let $\lambda \to +\infty$, then $\lambda y \to +\infty$. This implies that b is *not* limitational and we have shown that limitationality $\Rightarrow$ nulljointness.

To prove the converse, nulljointness $\Rightarrow$ limitationality, assume that y and b are nulljoint and let

$(y^o, b^o) \in P(x)$,
with $y^o > 0$. By nulljointness, $b^o > 0$.
Consider

$(\lambda y^o, \lambda b^o) \in \lambda P(x) = P(\lambda x), \lambda > 0$,
with the bound $\bar{b}$ on the undesirable output. Then there exists a $\hat{\lambda}$ such that

$\hat{\lambda} b^o = \bar{b}$, i.e., the bad restriction is binding.

Thus $\hat{\lambda} = \bar{b}/b^o$ implying that

$\hat{\lambda} y^o = (\bar{b}/b^o) y^o < +\infty$.

Thus b is limitational.

QED

This theorem tells us that under constant returns to scale (as in long run equilibrium) if the undesirable output b is quantity-regulated, i.e., $b \leqq \bar{b}$, then the desirable output is bounded given that the two outputs are nulljoint.

This undesirable result can be at least partially circumvented by introducing technical change, so that 'more good can be produced with the given bad'. In terms of our example, we would introduce a technical change component, say $h(t)$, such that

$$P(x,t) = \{(y,b) : \quad y + (b - h(t)y) \leqq x \quad \text{if } (b - h(t)y) \geqq 0 \\ 0 \text{ otherwise}\}, \tag{4}$$

with $h(t)$ decreasing over time. This might be called 'green' technical change.

2 References

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