Inertia risk: accounting for delayed impacts in catastrophic risk structures and implications for policy

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Motivation

- Increasing interest in catastrophic risk
  – Climate change
  – Ecological disasters
- Economic analysis makes assumptions about the structure of dynamic risk
- We will argue that these assumptions are inherently unrealistic for most interesting problems and suggest a possible improvement

Background

- Dynamic Economic analysis of catastrophic risk will usually frame the risk dynamics in terms of a hazard rate. If *f*(*y*) is the pdf and F(y) is the cdf, the definition is:

$$\lambda(y) = \lim_{h \to 0} \frac{\Pr\left(Y \in [y, y+h] \mid Y \geq y\right)}{h} = \frac{f(y)}{1 - F(y)}$$

- Well known result: Under weak conditions, any piecewise differentiable hazard rate $\lambda(y)$ gives rise to a pdf, *f*(*y*) and a cdf, *F*(*y*)

$$f(y) = \lambda(y)\exp\left(-\int_{y_0}^{y} \lambda(\eta)\,d\eta\right) \text{ and } F(y) = 1 - \exp\left(-\int_{y_0}^{y} \lambda(\eta)\,d\eta\right)$$

- We can specify a suitable hazard rate and be sure that there is a corresponding pdf/cdf

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Existing Models Of Catastrophic Risk

- Exogenous vs Endogenous Risk.
  - With Exogenous risk, the hazard rate is constant or depends on time
- Endogenous risk is the case when the hazard rate depends on state and/or control variables. There are two types
  - Time Distributed Catastrophe (TDC)
  - State Space Distributed Catastrophe (SDC)

Time Distributed Catastrophes

- By far the most common approach to modelling catastrophic risk.
- Usually takes a hazard rate $\lambda_t(x(t))$ as the starting point. Any path $x(t)$ gives rise to the pdf:

$$f(t) = \lambda_t(x(t)) \exp\left(-\int_0^t \lambda_t(x(\eta))\, d\eta\right)$$

- The problem: If we are able to freeze $x(t)$ then the hazard rate is a constant forever. The catastrophe will occur with probability one.

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State Space Distributed Catastrophes

- Starts out with a specification of some random variable distributed in state space.

$$X \sim h(x)$$

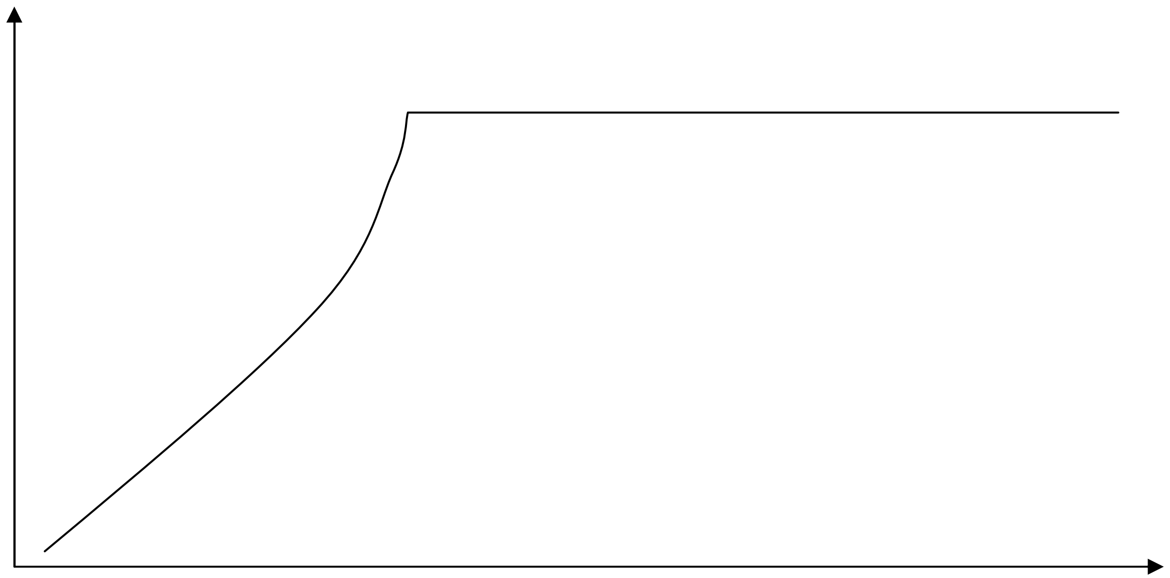
- Then one can construct a hazard rate

$$\lambda_x(x) = \max\left(\frac{h(x)}{1-H(x)}\frac{dx}{dt}, 0\right)$$

- The problem: If we are able to freeze x(t) then the hazard rate, instantly becomes zero

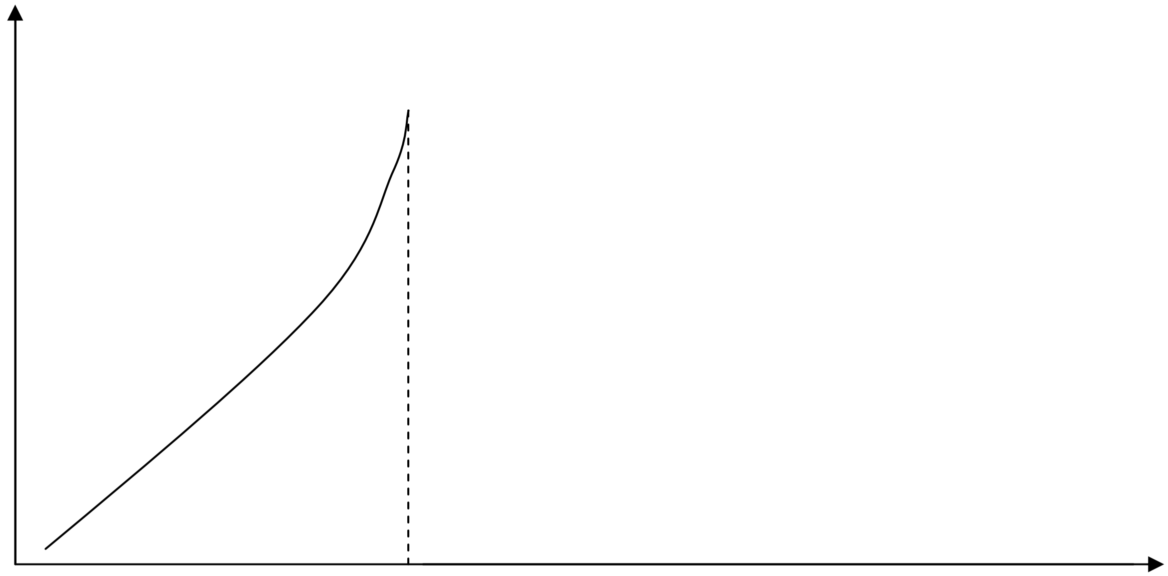
Thought experiment - TDC

- Let temperature increase at a fixed rate and then stabilize at a given level. Standard assumptions about hazard rates gives the following time path for the hazard rate



Thought experiment - SDC

- If we have a SDC and have the same scenario, the hazard rate looks like this:



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Implications

- TDCs, the most common approach does not solve a problem of how much risk we should accept. It solves the problem: Given that we will have a catastrophe, what is the optimal expected waiting time for this event to occur.
- Hardly a framework for sustainable resource management
- SDCs on the other hand seems too optimistic. If we're in a steady state, the catastrophe will never occur

Our solution – Inertia Risk

- We model risk as a set of cascading differential equations triggering an event
- To illustrate by example:
- Let *u* be emissions, *x* be stock of some pollutant and let *s* be environmental ”stress”.

$$\dot{x} = u(t) - \delta x$$

$$\dot{s} = \alpha x - \gamma s$$

- Let a catastrophe be triggered if $s > S \sim h(s)$.
- Assume the hazard rate associated with $h(s)$ is give by $\lambda \times s$. (Implies $h(s) = \lambda s \mathrm{Exp}(-\lambda s^2)$)

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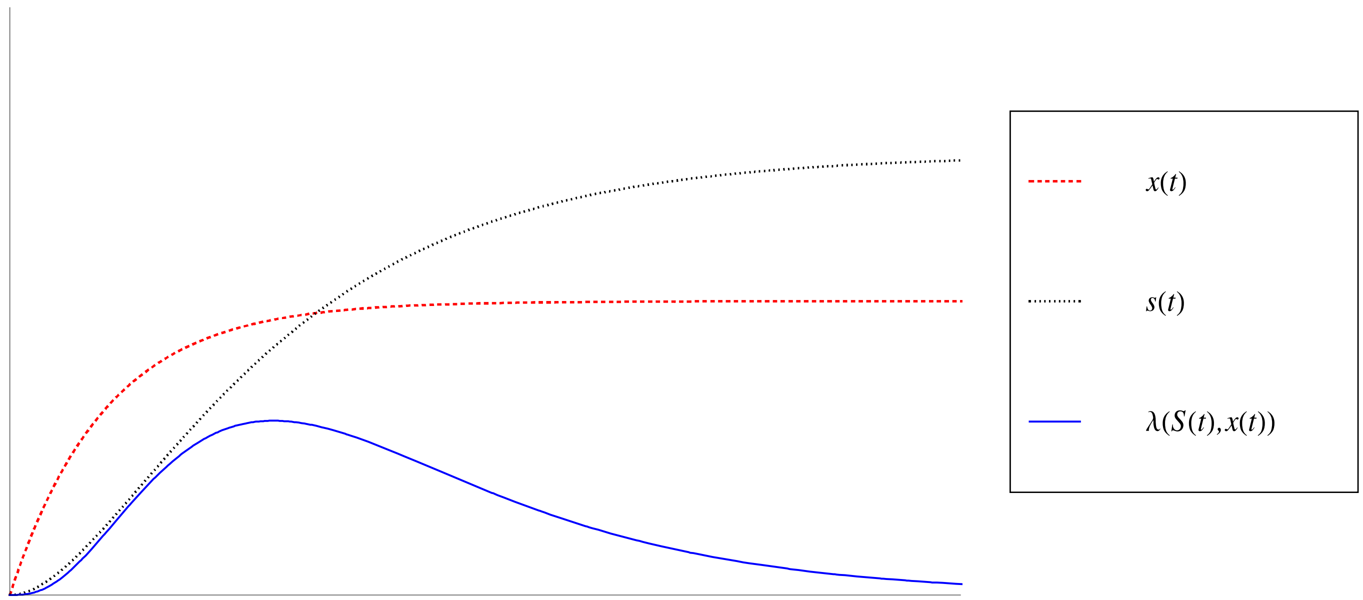
Risk under a constant emission rate



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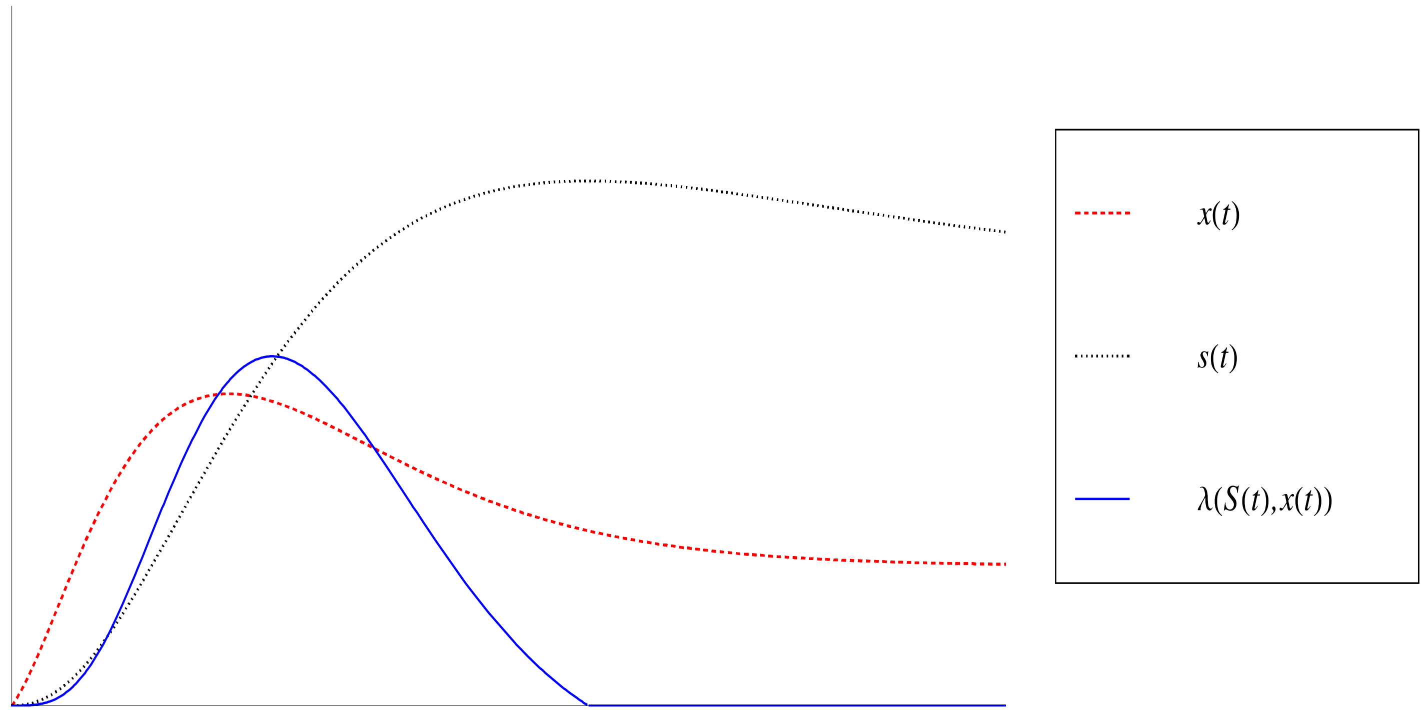
Risk with a pulse emission



An example

- Assume that temperatures are determined by:

$$\dot{x} = u - \delta x + \beta \frac{s^{\eta}}{s^{\eta} + a^{\eta}}$$

- We examine the special case $\eta \rightarrow \infty$ and assume that $\beta = 0$ for $s < S$ and $\beta = 0$ for $s > S$. Leads to a model where:

$$\dot{x} = \begin{cases} u - \delta x \text{ for } s < S \\ u - \delta x + \beta \text{ for } s > S \end{cases}$$

An example contd.

- Assume S ~ Exp(λ).
- Assume $\dot{s} = \alpha x - \gamma s$
- Assume linear damages $A \times x$ and quadratic abatement costs $\frac{c}{2}\left(u^0 - u\right)^2$

- We then solve the following problem:

Problem formulation

$$\max_{u(t)\geq 0} \mathbb{E}_{\tau}\left(\int_0^{\infty} \left(-Ax - \tfrac{c}{2}\left(u^0 - u\right)^2 \right) e^{-rt} dt \right)$$

$$\dot{x} = u - \delta x + B,\ \ x(0) \text{ given}$$

$$\dot{s} = \alpha x - \gamma s,\ \ s(0) \text{ given}$$

$$\dot{B} = 0,\ B(0) = 0$$

$$\lim_{dt \to 0} \frac{\Pr\left(\tau \in [t, t+dt] \mid \tau \geq t\right)}{dt} = \lambda \max(\dot{s}, 0)$$

$$B\left(\tau^+\right) - B\left(\tau^-\right) = \beta$$

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Solution

- Three important pairs:
- (x_{ss}, s_{ss}) – The optimal steady state that the system converges to if we start regulating with $(x(0), s(0)) = (0,0)$
- $(x(\infty)_{\beta=0}, s(\infty)_{\beta=0})$ – The optimal steady state if there is no risk
- $(x(\infty)_{\beta>0}, s(\infty)_{\beta>0})$ – The optimal steady state after the catastrophe has happened.

Optimal Regulation Starting from the Origin, 1

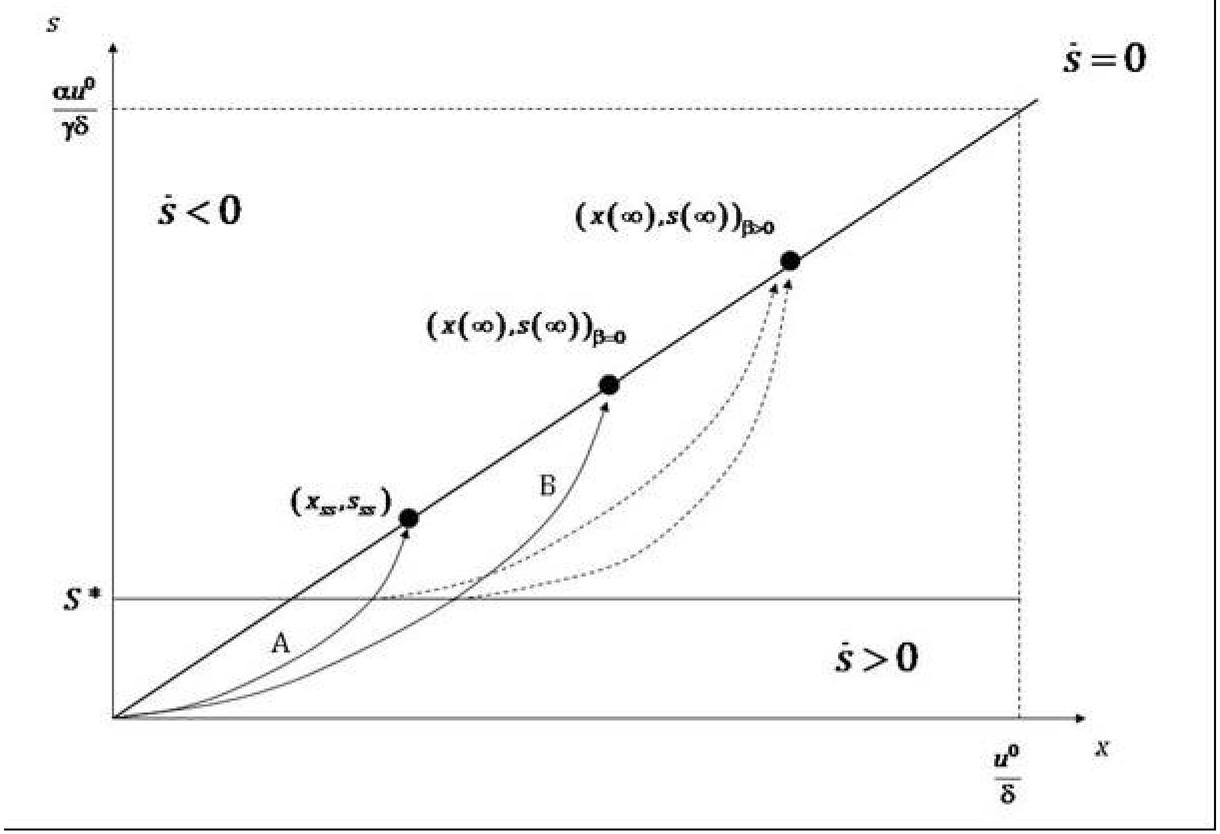

Diagram of paths in (x, s)-space illustrates two paths. One, A, which is the optimal policy where threshold risk is taken into account and a path B that does not take risk into account. In both cases, the threshold S is located so low that the threshold is crossed regardless. The dashed lines indicate optimal paths after stress threshold has been reached.*

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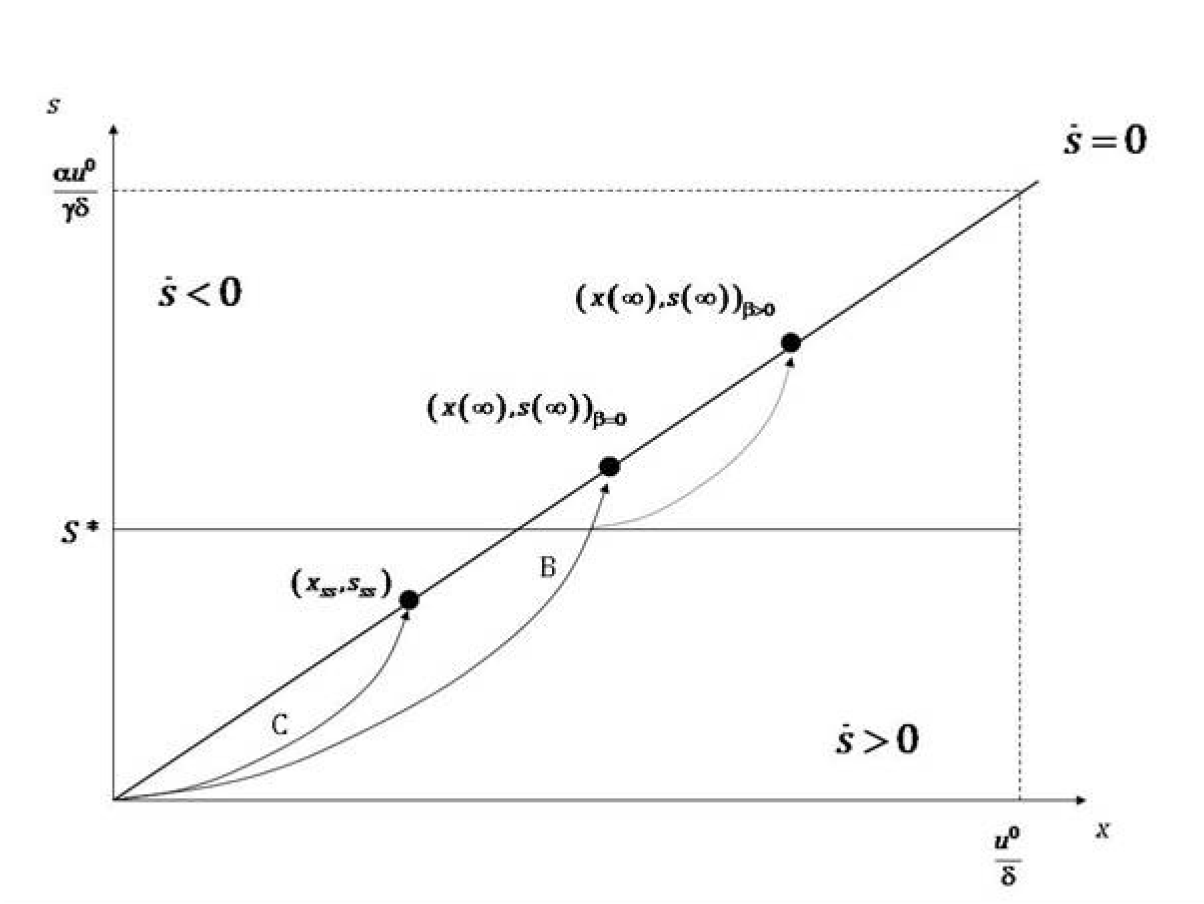
Optimal Regulation Starting from the Origin, 2



Here the threshold, S, is higher, so that if optimal policy, A, is followed, the catastrophe is not triggered. However, if a policy is followed that disregards risk, B, the threshold is crossed and the system will converge to* $(x(\infty)_{\beta>0}, s(\infty)_{\beta>0})$

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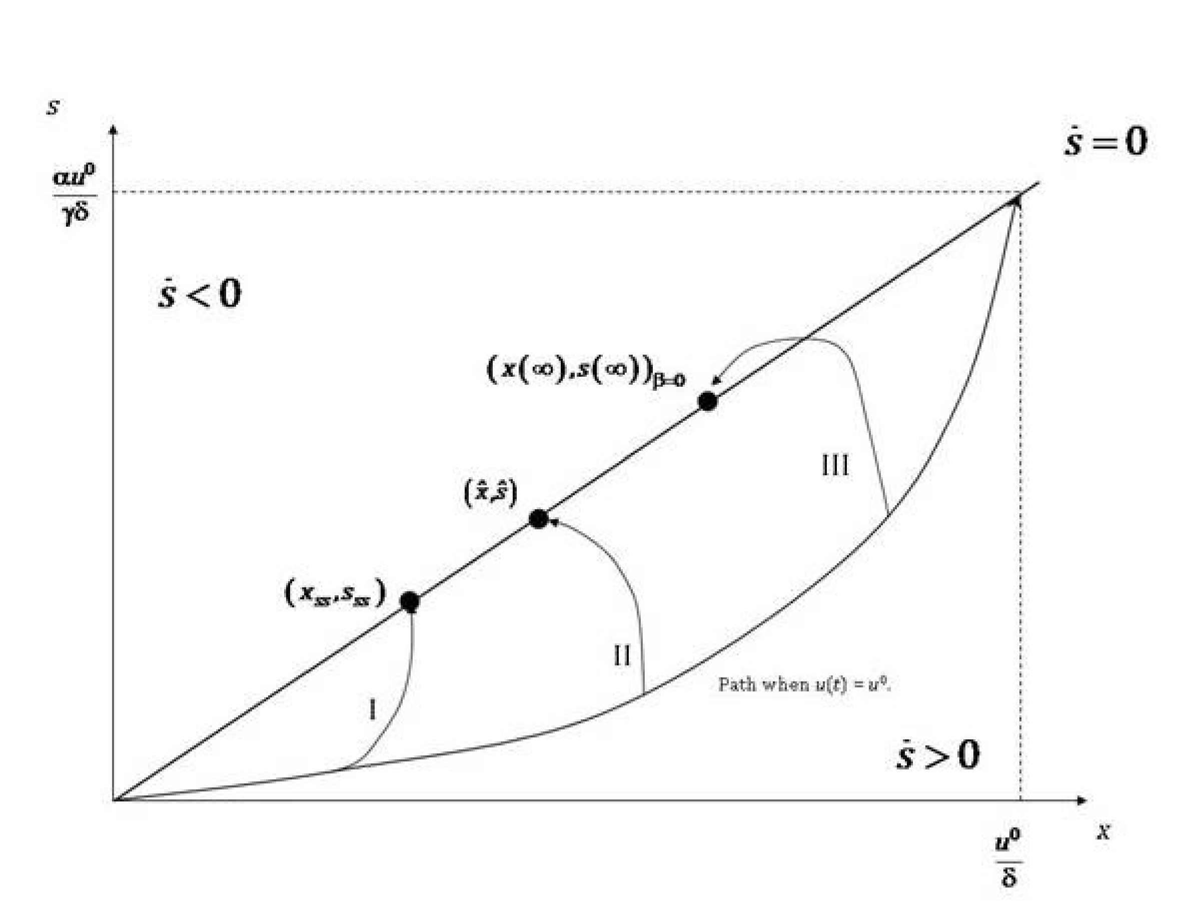
Optimal Paths not starting from the origin – Path Dependency



There are three paths that are optimal given different initial conditions. The paths are optimal as long as the threshold is not crossed

Comparison Between Inertia Risk And Time Distributed Catastrophes

- Here we compare apples and oranges. The reason we do this is that clever people have suggested that our concerns about the properties of TDCs are not important enough that we need this new framework.
- We beg to disagree
- In principle, if a model has properties that makes little sense, you should change it.

Steady state solution with Inertia Risk

$$x_{ss} = \frac{u^0}{\delta} - \frac{A}{c\delta(r+\delta)} + \underbrace{\frac{(r+\gamma)(r+\delta) - \sqrt{\left((r+\gamma)^2(r+\delta)^2 + \frac{2A\alpha^2\beta\lambda^2}{c(r+\delta)}\right)}}{\alpha\delta\lambda}}_{\text{Abatement because of threshold inertia risk}}$$

Note that the effect of abatement does not depend on u0. Further, if the system is relatively slow in that δ and γ are small numbers, then $(r + \delta)(r + \gamma)$ will also be a small number and the steady state abatement level will tend to be dominated by the terms with *A* in them. For large *A* and/or small $(r + \delta)$, abatement will be almost linear in *A*, α, β and λ. Obviously, if the term $A\alpha\lambda / c(r + \delta)$ becomes sufficiently large, then $x_{ss} \leq 0$. When this happens it is optimal to not let *x(t)* increase above 0 at all.

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Time Distributed Risk

- One can here also calculate the steady state. It looks like this:

$$x_{ss}^{TDC} = \frac{u^0}{\delta} - \frac{B - \delta\sqrt{c(r+\delta)\left(cr^2(r+\delta)C + \lambda(c\delta(r+\delta)(2r+\delta)+2A\lambda)D\right)}}{E}$$

$$B = c^2\delta(r+\delta)^3(r\delta + u^0\lambda) + 2Ac(r+\delta)\lambda(r\delta + u^0\lambda)$$

$$C = (2A\lambda + c(r+\delta)(\delta(r+\delta) - u^0\lambda))^2$$

$$D = \left(\lambda A^2 - 2c(r+\delta)(r(r+\delta) + (U+\beta)\lambda)A + c^2u^0(r+\delta)^2(2r(r+\delta) + u^0\lambda)\right)$$

$$E = c\delta(r+\delta)\lambda(c\delta(r+\delta)(2r+\delta) + 2A\lambda)$$

- I have no idea how to interpret this result, but they are quite different

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The End...

- Thank you for your time.
- I hope I have been able to convince you that this stuff is important.