When financial imperfections are not the problem, but the solution

Maria Arvaniti [1] Andrés Carvajal [2]

[1] CERE, Umeå Universitet

[2] University of Western Ontario and EPGE–FGV

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Motivation: the Gulf of Mexico disaster

Figure : Deepwater Horizon

Motivation: the Gulf of Mexico disaster

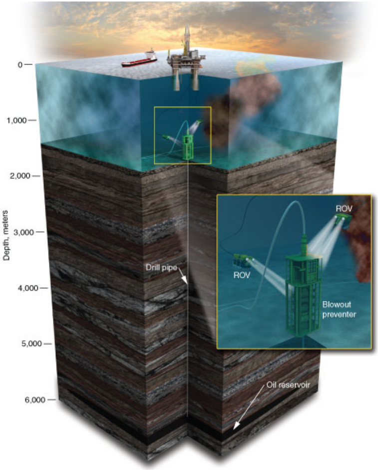

Figure : The Macondo Well

Motivation: the Gulf of Mexico disaster

Figure : Deepwater Horizon in the Gulf

Motivation: the Gulf of Mexico disaster

Figure : Deepwater Horizon on April 20th, 2010

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- eleven members of the rig's crew dead;
- 87 days of crude oil spill:
  - 4.9M barrels;
  - the largest in the history of petroleum production;
  - the worst environmental catastrophe in US history.

Motivation: the Gulf of Mexico disaster

Less immediate environmental consequences:

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- stranding of local sea turtles multiplied five-fold;
- one half of local dolphins with serious health disorders;
- long-run seabird deaths between 600,000 and 800,000.

Motivation: the Gulf of Mexico disaster

Less immediate economic consequences:

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- one third of tourism bookings cancelled or postponed;

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Less immediate economic consequences:

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- over 88,000Km^2 closed to *all* fishing;
- one third of tourism bookings cancelled or postponed;
- significant decrease in demand for local seafood.

Motivation: the Gulf of Mexico disaster

Less immediate well-being consequences:

- 25% increase in diagnosis of depression;

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- 25% increase in diagnosis of depression;
- over one half of local population reported to be “almost constantly” worried.

Motivation: the Gulf of Mexico disaster

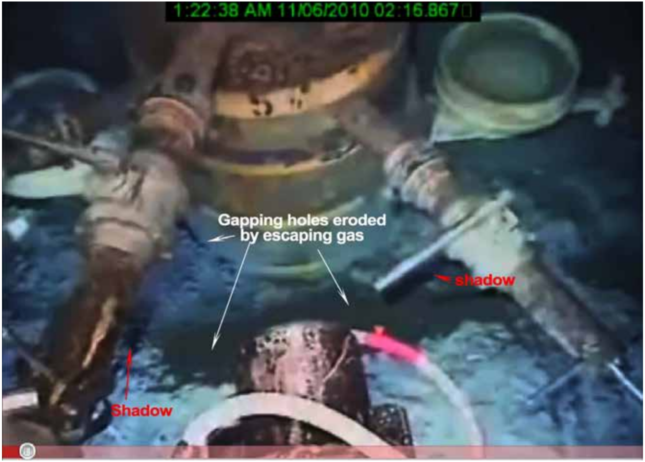

Figure : This could have been prevented: failures on the concrete seal

Motivation: the Gulf of Mexico disaster

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- cost USD14B;
- captured only 25% of the spill;
- 25% remains as residue;
- 50% dispersed from the waters of the gulf.

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The impact on BP:

- an immediate escrow of USD20B;
- it covered the clean-up cost (USD14B);
- it was fined USD18B by a US Federal Court;
- it stopped cash dividends for three quarters;
- it lost USD17B in market cap.

Motivation: the Gulf of Mexico disaster

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- BP has appealed the fine it received.

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- Liability insurance
  - mandatory liability insurance around $500 million much lower than estimated damages of an oil spill
  - large oil firms like BP are “self-insured" for catastrophic events through their **diversified investment portfolio**

Research Question

In light of the previous analysis, I ask the following questions:

- **Is it possible to create incentives for firms to prevent such catastrophic events through an intervention in the assets markets?**
- **In a context of complete or incomplete assets markets, does there exist a Pareto Improving reallocation of existing assets?**

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- endowments e_s^i;

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- complete elementary securities.

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- two states of the world, $s = 1, 2$;
- a single commodity;
- endowments e_s^i;
- complete elementary securities.
- securities holdings are θ_s^i with price q_s

Endogenous probabilities (Here comes BP!)

We depart from the classical model:

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$$c_0 + \pi(\varepsilon) \cdot u_1^i(c_1) + [1 - \pi(\varepsilon)] \cdot u_2^i(c_2);$$

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$$c_0 + \pi(\varepsilon) \cdot u_1^i(c_1) + [1 - \pi(\varepsilon)] \cdot u_2^i(c_2);$$

- effort subtracts from agent 0's present consumpion:

  trade-off

Two assumptions

For a well-behaved problem:

- interiority: for some $\hat{\varepsilon}$,

$$\pi'(\hat{\varepsilon}) = \frac{1}{u_1^0(\hat{c}_1^0) - u_2^0(\hat{c}_2^0)},$$

  where $(\hat{c}_s^i)_{i=0}^I$ maximizes

$$\sum_{i=0}^{I} u_s^i(c^i) \ : \ \sum_{i=0}^{I} c^i = \sum_{i=0}^{I} e_s^i.$$

Two assumptions

For a well-behaved problem:

- concavity: the following matrix is negative definite:

$$\begin{pmatrix} \pi(\hat{\varepsilon}) \cdot \partial^2 u_1^0(\hat{c}_1^0) & 0 & \pi'(\hat{\varepsilon}) \cdot \partial u_1^0(\hat{c}_1^0) \\ 0 & [1 - \pi(\hat{\varepsilon})] \cdot \partial^2 u_2^0(\hat{c}_2^0) & -\pi'(\hat{\varepsilon}) \cdot \partial u_2^0(\hat{c}_2^0) \\ \pi'(\hat{\varepsilon}) \cdot \partial u_1^0(\hat{c}_1^0) & -\pi'(\hat{\varepsilon}) \cdot \partial u_2^0(\hat{c}_2^0) & \pi''(\hat{\varepsilon}) \cdot [u_1^0(\hat{c}_1^0) - u_2^0(\hat{c}_2^0)] \end{pmatrix}.$$

Competitive equilibrium

Nash-Walras equilibrium is $\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\}$ such that:

1. $(\bar{\vartheta}^0, \bar{\varepsilon})$ maximizes

$$-\varepsilon - \bar{q} \cdot \vartheta^0 + \pi(\varepsilon) \cdot u_1^0(e_1^0 + \vartheta_1^0) + [1 - \pi(\varepsilon)] \cdot u_2^0(e_2^0 + \vartheta_2^0);$$

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2. each $\bar{\vartheta}^i$ maximizes

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3. and securities markets clear:

$$\sum_{i=0}^{I} \bar{\vartheta}^i = 0.$$

Competitive equilibrium

Nash-Walras equilibrium is characterized by:

1. the first-order conditions of 0:

$$1 = \pi'(\bar{\varepsilon}) \cdot [u_1^0(e_1^0 + \bar{\vartheta}_1^0) - u_2^0(e_2^0 + \bar{\vartheta}_2^0)],$$

$$\bar{q}_1 = \pi(\bar{\varepsilon}) \cdot \partial u_1^0(e_1^0 + \bar{\vartheta}_1^0) \text{ and } \bar{q}_2 = [1 - \pi(\bar{\varepsilon})] \cdot \partial u_2^0(e_2^0 + \bar{\vartheta}_2^0);$$

2. the first-order conditions of each $i \geqslant 1$:

$$\bar{q}_1 = \pi(\bar{\varepsilon}) \cdot \partial u_1^i(e_1^i + \bar{\vartheta}_1^i) \text{ and } \bar{q}_2 = [1 - \pi(\bar{\varepsilon})] \cdot \partial u_2^i(e_2^i + \bar{\vartheta}_2^i).$$

3. and the market clearing condition:

$$\sum_{i=0}^{I} \bar{\vartheta}^i = 0.$$

Competitive equilibrium

Nash-Walras equilibrium is characterized by:

$$\mathcal{F}(q, \vartheta, \varepsilon) = \begin{pmatrix} \pi(\varepsilon) \cdot \partial u_1^0(e_1^0 + \vartheta_1^0) - q_1 \\ [1 - \pi(\varepsilon)] \cdot \partial u_1^0(e_2^0 + \vartheta_2^0) - q_2 \\ \left[u_1^0(e_1^0 + \vartheta_1^0) - u_2^0(e_2^0 + \vartheta_2^0)\right] \cdot \pi'(\varepsilon) - 1 \\ \pi(\varepsilon) \cdot \partial u_1^1(e_1^1 + \vartheta_1^1) - q_1 \\ [1 - \pi(\varepsilon)] \cdot \partial u_2^1(e_2^1 + \vartheta_2^1) - q_2 \\ \vdots \\ \pi(\varepsilon) \cdot \partial u_1^I(e_1^I + \vartheta_1^I) - q_1 \\ [1 - \pi(\varepsilon)] \cdot \partial u_2^I(e_2^I + \vartheta_2^I) - q_2 \\ \sum_{i=0}^{I} \vartheta^i \end{pmatrix} = 0.$$

Pareto efficiency

Given quasilinearity, social welfare:

- is

$$W = -\varepsilon + \sum_{i=0}^{I} \{\pi(\varepsilon) \cdot u_1^i(c_1^i) + [1 - \pi(\varepsilon)] \cdot u_2^i(c_2^i)\};$$

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- is maximized when

$$\pi'(\varepsilon) \cdot \sum_{i=0}^{I} [u_1^i(c_1^i) - u_2^i(c_2^i)] = 1;$$

  and, for each $i, j = 0, \ldots, I$,

$$\partial u_s^i(c_s^i) = \partial u_s^j(c_s^j)$$

  for both $s = 1, 2$.

Pareto efficiency

- pareto inefficient level of effort: externality through the effect of effort on the probabilty of the state

Constrained inefficiency

Allocation (ε, c) is *weakly constrained inefficient* if there exist

$$[\tilde{\varepsilon}, \tilde{q}, (\tilde{\vartheta}^i, \tau^i)_{i=0}^I]$$

such that:

1. $\tilde{\varepsilon}$ maximizes

$$-\varepsilon + \pi(\varepsilon) \cdot u_1^0(e_1^0 + \tilde{\vartheta}_1^0) + [1 - \pi(\varepsilon)] \cdot u_2^0(e_2^0 + \tilde{\vartheta}_2^0);$$

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2. for each $i \geqslant 1$, $\tilde{\vartheta}^i$ maximizes

$$-\tilde{q} \cdot \vartheta^i + \pi(\tilde{\varepsilon}) \cdot u_1^i(e_1^i + \vartheta_1^i) + [1 - \pi(\tilde{\varepsilon})] \cdot u_2^i(e_2^i + \vartheta_2^i);$$

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3. $\sum_{i=0}^I \tilde{\vartheta}^i = 0$ and $\sum_{i=0}^I \tau^i = 0$;

Constrained inefficiency

Allocation (ε, c) is *weakly constrained inefficient* if there exist

$$[\tilde{\varepsilon}, \tilde{q}, (\tilde{\vartheta}^i, \tau^i)_{i=0}^I]$$

such that:

4. every $i \geqslant 1$ is better-off:

$$-\tilde{q}_1 \cdot \tilde{\vartheta}_1^i - \tilde{q}_2 \cdot \tilde{\vartheta}_2^i + \tau^i + \pi(\tilde{\varepsilon}) \cdot u_1^i(e_1^i + \tilde{\vartheta}_1^i) + [1 - \pi(\tilde{\varepsilon})] \cdot u_2^i(e_2^i + \tilde{\vartheta}_2^i)$$
$$> c_0^i + \pi(\varepsilon) \cdot u_1^i(c_1^i) + [1 - \pi(\varepsilon)] \cdot u_2^i(c_2^i);$$

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$$> c_0^i + \pi(\varepsilon) \cdot u_1^i(c_1^i) + [1 - \pi(\varepsilon)] \cdot u_2^i(c_2^i);$$

5. 0 is better-off:

$$-\tilde{\varepsilon} - \tilde{q}_1 \cdot \tilde{\vartheta}_1^0 - \tilde{q}_2 \cdot \tilde{\vartheta}_2^0 + \tau^0 + \pi(\tilde{\varepsilon}) \cdot u_1^0(e_1^0 + \tilde{\vartheta}_1^0) + [1 - \pi(\tilde{\varepsilon})] \cdot u_2^0(e_2^0 + \tilde{\vartheta}_2^0)$$
$$> -\varepsilon + c_0^0 + \pi(\varepsilon) \cdot u_1^0(c_1^0) + [1 - \pi(\varepsilon)] \cdot u_2^0(c_2^0).$$

Constrained inefficiency

If such

$$[\tilde{\varepsilon}, \tilde{q}, (\tilde{\vartheta}^i, \tau^i)_{i=0}^I]$$

exists with $\tau^0 = 0$, allocation (ε, c) is *strongly constrained inefficient.*

Constrained policy intervention

Around $\{\varepsilon, \vartheta, q\}$, a perturbation $d\vartheta^0$ induces

$$\{d\varepsilon, (d\vartheta^i)_{i \neq 0}, dq\}$$

via:

- the first-order condition

$$1 = \pi'(\varepsilon) \cdot [u_1^0(e_1^0 + \vartheta_1^0) - u_2^0(e_2^0 + \vartheta_2^0)];$$

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- every *other* first-order condition

$$q_1 = \pi(\varepsilon) \cdot \partial u_1^i(e_1^i + \vartheta_1^i) \text{ and } q_2 = [1 - \pi(\varepsilon)] \cdot \partial u_2^i(e_2^i + \vartheta_2^i);$$

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- market clearing, $\sum_{i=0}^{I} \vartheta^i = 0$;

Weak constrained inefficiency

Equilibrium

$$\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\}$$

is *constrained inefficient* if there exists a policy $d\vartheta^0$ that induces

$$dW > 0$$

via

$$\{d\varepsilon, (d\vartheta^i)_{i=0}^I\}.$$

Generic weak constrained inefficiency of equilibrium

Around equilibrium

$$\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\},$$

dW is the sum of

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1. $-d\varepsilon$;
2. $$\sum_{i=0}^{I} [u_1^i(\bar{c}_1^i) - u_2^i(\bar{c}_2^i)] \cdot \pi'(\bar{\varepsilon}) \cdot d\varepsilon;$$

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2. $$\sum_{i=0}^{I} [u_1^i(\bar{c}_1^i) - u_2^i(\bar{c}_2^i)] \cdot \pi'(\bar{\varepsilon}) \cdot d\varepsilon;$$
3. and
$$\pi(\bar{\varepsilon}) \cdot \sum_{i=0}^{I} \partial u_1^i(\bar{c}_1^i) \cdot dc_1^i + [1 - \pi(\bar{\varepsilon})] \cdot \sum_{i=0}^{I} \partial u_2^i(\bar{c}_2^i) \cdot dc_2^i.$$

Generic weak constrained inefficiency of equilibrium

Around equilibrium

$$\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\},$$

by direct computation,

$$dW = \sum_{i=1}^{I} [u_1^i(\bar{c}_1^i) - u_2^i(\bar{c}_2^i)] \cdot \pi'(\varepsilon) \cdot d\varepsilon.$$

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On the other hand,

$$d\varepsilon = \frac{\pi'(\varepsilon) \cdot \left[\partial u_2^0(\bar{c}_2^0) \cdot d\vartheta_2^0 - \partial u_1^0(\bar{c}_1^0) \cdot d\vartheta_1^0\right]}{\pi''(\varepsilon) \cdot [u_1^0(\bar{c}_1^0) - u_2^0(\bar{c}_2^0)]}.$$

Generic weak constrained inefficiency of equilibrium

- direction of the Pareto improving policy depends on the sign of

$$\sum_{i=1}^{I}[u_1^i(\bar{c}_1^i) - u_2^i(\bar{c}_2^i)].$$

  - if positive, $d\epsilon$ must be positive too: competitive level of ϵ inefficiently low, $d\theta_2^0 < 0$ and $d\theta_1^0 > 0$
  - if negative, $d\epsilon$ negative, competitive level of ϵ inefficiently high, $d\theta_2^0 > 0$ and $d\theta_1^0 < 0$

Generic weak constrained inefficiency of equilibrium

- $\sum_{i=1}^{I}[u_1^i(\bar{c}_1^i) - u_2^i(\bar{c}_2^i)] \neq 0$ generically in the space of endowments so that there is almost always a Pareto improving policy.

Theorem

For any $\{u, \pi\}$, the competitive equilibrium allocation is weakly constrained inefficient, generically on e.

Strong constrained inefficiency

Equilibrium

$$\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\}$$

is *strongly constrained inefficient* if there exists a policy $d\vartheta^0$ that induces

$$dU^0 > 0$$

and

$$\sum_{i=1}^{I} dU^i > 0,$$

via

$$\{d\varepsilon, (d\vartheta^i)_{i=0}^{I}\}.$$

Three more assumptions

We now assume, furthermore:

- catastrophe:

$$u_1^0(\hat{c}_1^0) - u_2^0(\hat{c}_2^0) > 0 \text{ and } \sum_{i=1}^{I} [u_1^i(\hat{c}_1^i) - u_2^i(\hat{c}_2^i)] > 0.$$

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- relief aid: an external source funds agents $i \geqslant 1$ in their purchases of ϑ_2^i, for a total of $\rho > 0$ units of the asset.

Three more assumptions

We now assume, furthermore:

- catastrophe:

$$u_1^0(\hat{c}_1^0) - u_2^0(\hat{c}_2^0) > 0 \text{ and } \sum_{i=1}^{I} [u_1^i(\hat{c}_1^i) - u_2^i(\hat{c}_2^i)] > 0.$$

- relief aid: an external source funds agents $i \geqslant 1$ in their purchases of ϑ_2^i, for a total of $\rho > 0$ units of the asset.

- heterogeneity: for $i, j \geqslant 1$, if $i \neq j$, then, for both $s = 1, 2$.

$$\partial^2 u_s^i(\hat{c}_s^i) \neq \partial^2 u_s^j(\hat{c}_s^j).$$

Strong constrained inefficiency

Equilibrium

$$\{\bar{\varepsilon}, \bar{\vartheta}, \bar{q}\}$$

is *strongly constrained inefficient* if there exists a policy $d\vartheta^0$ that induces positive total differentials for

$$-\varepsilon - q_1 \cdot \vartheta_1^0 - q_2 \cdot \vartheta_2^0 + \pi(\varepsilon) \cdot u_1^0(e_1^0 + \vartheta_1^0) + [1 - \pi(\varepsilon)] \cdot u_2^0(e_2^0 + \vartheta_2^0)$$

and

$$-q_1 \cdot \sum_{i=1}^{I} \vartheta_1^i - q_2 \cdot \left(\sum_{i=1}^{I} \vartheta_2^i - \rho\right) + \pi(\varepsilon) \cdot \sum_{i=1}^{I} u_1^i(e_1^i + \vartheta_1^i) + [1 - \pi(\varepsilon)] \cdot \sum_{i=1}^{I} u_2^i(e_2^i + \vartheta_2^i),$$

via

$$\{d\varepsilon, (d\vartheta^i)_{i=0}^{I}\}.$$

Generic weak constrained inefficiency of equilibrium

Around $\{e, u, \pi\}$, at equilibrium, the Jacobean of

$$(q, \vartheta, \varepsilon) \mapsto \mathcal{H} = \begin{pmatrix} U^0 \\ \sum_{i=1}^{I} U^i \\ \left[u_1^0(e_1^0 + \vartheta_1^0) - u_2^0(e_2^0 + \vartheta_2^0)\right] \cdot \pi'(\varepsilon) - 1 \\ \pi(\varepsilon; \delta_0) \cdot \partial u_1^1(e_1^1 + \vartheta_1^1) - q_1 \\ [1 - \pi(\varepsilon)] \cdot \partial u_2^1(e_2^1 + \vartheta_2^1) - q_2 \\ \vdots \\ \pi(\varepsilon; \delta_0) \cdot \partial u_1^I(e_1^I + \vartheta_1^I) - q_1 \\ [1 - \pi(\varepsilon; \delta_0)] \cdot \partial u_2^I(e_2^I + \vartheta_2^I) - q_2 \\ \sum_{i=0}^{I} \vartheta^i \end{pmatrix},$$

has full row rank.

Generic weak constrained inefficiency of equilibrium

Theorem

The competitive equilibrium allocation is strongly constrained inefficient, generically on $\{e, u, \pi\}$.

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Suppose now:

- individuals of types $i = 0, \ldots, I$;

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- given ε, within each type,
  - $\pi(\varepsilon)$ of the individuals are in state 1, while
  - $\frac{1}{2}[1 - \pi(\varepsilon)]$ are in states 2 and 3.

Uninsurable idiosyncratic risk

Suppose also that

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$$-\varepsilon - q \cdot b + \pi(\varepsilon) \cdot u^0(e^0 + b) + \frac{1}{2} \cdot [1 - \pi(\varepsilon)] \cdot [u^0(e^0 + z^0 + b) + u^0(e^0 - z^0 + b)],$$

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- and, for $i \neq 0$,

$$-q \cdot b + \pi(\varepsilon) \cdot u^i(e^i + b) + \frac{1}{2} \cdot [1 - \pi(\varepsilon)] \cdot [u^i(e^i + z^i + b) + u^i(e^i - z^i + b)].$$

Uninsurable idiosyncratic risk: competitive equilibrium

Nash-Walras equilibrium is characterized by:

1. the first-order conditions of 0:

$$1 = \pi'(\bar{\varepsilon}) \cdot \left\{ u^0(\bar{c}^0) - \tfrac{1}{2} \cdot \left[u^0(\bar{c}^0 + z^0) + u^0(\bar{c}^0 - z^0) \right] \right\}$$

and

$$\bar{q} = \pi(\bar{\varepsilon}) \cdot \partial u^0(\bar{c}^0) + \frac{(1 - \pi(\bar{\varepsilon}))}{2} \cdot \left[\partial u^0(\bar{c}^0 + z^0) + \partial u^0(\bar{c}^0 - z^0) \right].$$

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3. and the market clearing condition: $\sum_{i=0}^{I} m^i \cdot \bar{b}^i = 0$.

Uninsurable idiosyncratic risk: policy effects

Around equilibrium $\{\bar{\varepsilon}, \bar{b}, \bar{q}\}$, policy db^0 induces dW equal to the sum of

1. $-d\varepsilon$;
2. $$\sum_{i=0}^{I} m^i \cdot \left\{u^i(\bar{c}_1^i) - \frac{1}{2} \cdot \left[u^i(\bar{c}_2^i) + u^i(\bar{c}_3^i)\right]\right\} \cdot \pi'(\bar{\varepsilon}) \cdot d\varepsilon;$$
3. $$\pi(\bar{\varepsilon}) \cdot \sum_{i=0}^{I} m^i \cdot \partial u^i(\bar{c}_1^i) \cdot dc^i;$$
4. and
$$\frac{1}{2} \cdot [1 - \pi(\bar{\varepsilon})] \cdot \sum_{i=0}^{I} m^i \cdot \left[\partial u^i(\bar{c}_2^i) + \partial u^i(\bar{c}_3^i)\right] \cdot dc^i.$$

Uninsurable idiosyncratic risk: generic inefficiency

Around equilibrium $\{\bar{\varepsilon}, \bar{b}, \bar{q}\}$,

- by direct computation,

$$dW = \sum_{i=1}^{I} m^i \cdot \left\{ u^i(\bar{c}_1^i) - \frac{1}{2} \cdot \left[u^i(\bar{c}_2^i) + u^i(\bar{c}_3^i) \right] \right\} \cdot \pi'(\bar{\varepsilon}) \cdot d\varepsilon;$$

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- while

$$d\varepsilon = -\frac{\pi'(\bar{\varepsilon})}{\pi''(\bar{\varepsilon})} \cdot \frac{\partial u^0(\bar{c}_1^0) - \frac{1}{2} \cdot \left[\partial u^0(\bar{c}_2^0) + \partial u^0(\bar{c}_3^0) \right]}{u^0(\bar{c}_1^0) - \frac{1}{2} \cdot \left[u^0(\bar{c}_2^0) + u^0(\bar{c}_3^0) \right]} \cdot db^0.$$

Uninsurable idiosyncratic risk: generic inefficiency

- CE level of effort is inefficiently low
  - for a concave marginal utility function, $db^0 > 0$: saving more in equilibrium is welfare improving for the whole society.
  - for a convex marginal utility function (prudency), $db^0 < 0$: agent 0 should save less in equilibrium.

- preventing an agent of type 0 from saving optimally, induces him to exert a higher level of effort since by doing so he makes the future look less "volatile"