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Managing Water: Rights, Markets, and Welfare*

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Abstract

This article investigates key aspects related to managing water resources, and explores their implications for economic efficiency: incomplete property rights; overallocation of water; the divergence between water entitlements and productivity of water use. These issues are explored in a production model with a single input, water drawn from a common source, and two main insights are offered. First, a novel result relating welfare and water entitlements is established, an equivalence between the socially optimal and legal assignment of water rights, via a set of social weights implied by the rights assignment. It is also shown that, for water entitlements for which the divergence between productivity and entitlements is substantial, no set of valid social priorities can lead to the socially efficient allocation. Second, considering a hypothetical water market with an *endogenous* price, it is found that trade in water is unable to eliminate allocative inefficiency, and that taxes have unexpectedly moderate effects on trade and welfare, with the majority of tax shifts canceled out by changes in demand. In addition, trade is not effective at facilitating efficiency-enhancing reallocation of water under scarcity when entitlements diverge substantially from productivity. The results here highlight important new connections between welfare and water entitlements and the limitations of market-based instruments under incomplete property rights, and have implications for designing property rights regimes for managing water under scarcity.

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1 Introduction

Efficient use of natural resources is crucial for long-term economic and social well-being, particularly in the face of increasing scarcity. Water is one such key natural resource, already viewed differently from most economic goods because it is necessary both for human existence and for many production activities. Additionally it has the property of being a so-called 'fugitive' resource, which often requires extraction or transportation to be put to valuable use, leading to water having special treatment under the law in many locales. Broadly speaking, water can be treated as private, public or common property, and it is common for legal systems around the world to mix all three (Schorr (2017)). Civil law countries tend to derive current water treatment from Roman law, allowing for distinct treatment of public and private waters. Common law treatment of water begins with riparian rights, which treat water as common property of all landowners adjacent to a body of water, later developing into permit-based and statutory systems (e.g. Australia, see Grafton and Horne (2014)) and prior appropriation rights (e.g. the western United States, see Brewer et al. (2006)).[1] Regardless of the legal specifics, the result is often a form of a queuing system.

Property rights generally attempt to address two economic criteria, "security" and "flexibility" (Ciriacy-Wantrup (1956) and Livingston (1995)). Security relates to water development, particularly how investment behavior will be affected by uncertainty, either in terms of physical availability of the resource or the tenure of allowed usage. Flexibility refers to water entitlement allocations, particularly limitations on voluntary and involuntary transfers, as well as additional factors such as the priority of new entrants. However, there exists a natural tension between these two criteria, with security often being deemed of greater importance in water-scarce regions (influencing the choices of appropriative rights in the western U.S. and statutory rights for Australia)[2] and flexibility in water-abundant ones (illustrated by the inherent proportionality of basic riparian common law traditions).

Many water allocation regimes, including the statutory ones of the Murray-Darling basin in Australia and those of prior appropriation in the Western U.S. states, view river waters as being largely a 'commons' and only assign so-called usufruct rights, subject to a broadly defined 'beneficial use' requirement, and often involve a variety of restrictions on the use, storage, and transfer of water, with permanent transfers rarely allowed easily.[3] Consequently, the property rights over water are usually termed *incomplete*. Property rights and their importance for efficient use, and optimal development, of natural resources have been explored in contexts as widely varying as fishing, mineral and timber

[1] See Schorr (2017) for a broader legal discussion of water rights around the world; Dellapenna (2004) for an overview of the legal treatment of surface water under the three major rights systems of the United States; and Young (2014) for a recent overview of water entitlement (i.e. legal) regimes around the world.

[2] Even within these broad categories there is variation in the nature of rights in water scarce regions. To illustrate, water rights differ even across the Western U.S. states where the prior appropriation doctrine is applicable, see Brewer et al. (2006); Johnson et al. (1981); Libecap (2010)).

[3] However, we note that in reality there is wide variation in what constitutes a transfer of rights, and in the impediments to such transfers, see e.g. Johnson et al. (1981), Gould (1989), Brewer et al. (2006), Libecap (2010) for the case of the Western U.S. and Grafton and Horne (2014) for the Australian case.

development, and surface water (e.g. Grafton et al. (2000); Libecap (2007)). The role of, and benefits from, flexible property rights regimes in general appear to depend upon the nature of the resource, with water, due to its multi-dimensional nature (e.g. often having qualities of both a public and private good), being particularly complex. The role of the incompleteness (inflexibility) of property rights related to water in limiting efficient reallocation of water across uses is difficult to either quantify conceptually or measure empirically (due to the absence of the counter-factual in the latter case, although see Libecap (2010)). However, these restrictions on property rights may be viewed as non-linear transaction costs, and they are what lead to the key role for pre-existing property rights structures in our analysis.

Property rights play an important role in determining economic efficiency, and together with productivity of use, drive patterns of water use. These aspects, thus, are intimately related to managing water in an increasingly dry world. Indeed, the necessity of managing water scarcity has led to substantial market-based reforms related to water rights in many regions of the world, including South Africa (Schreiner (2015)), Australia (Grafton and Horne (2014)), and Chile (Donoso (2015)). Major episodes of scarcity have occasionally led to substantial rethinking of existing property rights structures in the context of water, e.g legal reforms as a part of market development in Australia (McKay (2005)). Shortages, particularly a pair of droughts in California, from 2007-2009 (Christian-Smith et al. (2011)) and again from 2012-2015 (Swain (2015)), have also been a major factor contributing to shifts in many water-related policies in California.[4] In any case, water rights play an important role in shaping water policy and dealing with water scarcity in many regions of the world.

Motivated by their importance in determining water use and their implications for managing scarcity, we aim to assess how legal structures related to water rights affect societal well-being. In essence, we analyze the efficiency-related aspects of water rights regimes, considering both previously unexplored theoretical properties of inflexible rights mechanisms as well as the extent to which trade in water may enable increases in welfare when such rights are present. The latter aspect is particularly interesting since a coherent, quantitative understanding of how pre-existing rights mechanisms affect water trade appears to be lacking. To this end, we develop a simple model for production where the only input is water drawn from a common source (e.g. a river), with which we seek to answer two complementary questions. First, how is the legal allocation of water related to the socially optimal allocation? Second, what are the quantitative implications of different relationships between water rights and productivity of water use? To what extent can trade lead to welfare increases under scenarios of water shortages? Our notion of optimality is a slight generalization of allocative efficiency ('efficiency'), where society may, for many reasons including concerns regarding equity, choose to prioritize water users differently.

[4]These include California Executive Order B-29-15 (https://www.gov.ca.gov/docs/4.1.15_Executive_Order.pdf), which mandates a 25% reduction in potable urban water usage state-wide, and the passage of the Sustainable Groundwater Management Act (http://groundwater.ca.gov/docs/2014%20Sustainable%20Groundwater%20Management%20Legislation%20with%202015%20amends%201-15-2016.pdf), with a mandate for local groundwater management plans.

Our analysis intersects with two strands of the vast literature on the economics of water. The first strand are studies dealing with certain aspects related to flexibility and efficiency of water rights (with a pre-dominant focus on the Western U.S., e.g. Burness and Quirk (1979, 1980a,b); Johnson et al. (1981); Howe et al. (1982); Weber (2001)) and is primarily theoretical, dealing largely with conceptual questions.[5] The second strand are studies concerned with various aspects of the market for water, rather than water rights, including ones dealing with transactions costs involved in such markets and the gains from trade in water (Diao et al. (2005); Ward and Pulido-Velázquez (2008); Libecap (2010); Brewer et al. (2008); Brookshire et al. (2004); Wong and Eheart (1983)). This strand of literature is primarily empirical or simulation-based, evaluating aspects of existing markets or simulating hypothetical markets.

Our analysis builds upon these two strands of literature and provides an integrated view of water rights and trading in water, two aspects that have not, to our knowledge, been jointly investigated. More explicitly, for some scenarios of policy interest, we consider both the implications of existing water rights frameworks for allocative efficiency that accounts for social priorities, as well as the performance of a market for water based upon existing water rights. In the former case, we use a welfare-theoretic perspective to evaluate how arbitrary water rights allocations relate to the choices of a benevolent social planner; while in the latter case, we are interested in quantification of key features of a hypothetical market with real world features, conditional on these rights allocations.

We develop a multi-producer model, with a common resource (water) as the only input, and an endogenous market structure. Our focus is on heterogeneous individual producers and our analysis of economic efficiency of the legal aspects of water is intended to highlight the difference between legal and economic efficiency (following e.g. Burness and Quirk (1980b)), consistent with viewing legal doctrines as being motivated by considerations of economic efficiency (Libecap (2007); Posner (2007); Kaplow and Shavell (2002)). There is no uncertainty regarding availability in our model, we leave aside questions of water storage and water quality, and our model speaks to a market-based scenario, with many price-taking users. While using simulation parameters that speak to the context of the Western U.S., our analysis is motivated by, and applicable to, management of water resources in many water management settings exhibiting the the following characteristics: rights, often historically determined, are fundamental to current water allocations; transferability of rights is limited, based largely upon legal considerations; markets for water are extant (or are under active consideration), largely to alleviate scarcity. Consequently, while the model we develop shares broad similarities to earlier work, it differs from them in many important ways.

The first question posed above is essentially a generalization of the question regarding

[5]There is also a sizeable, primarily theoretical, literature related, often tangentially, to water rights, focusing on issues such as optimal infrastructure investment, storage, and delivery decisions (e.g. Guise and Flinn (1970), Brown et al. (1990) and Chakravorty et al. (1995)), and transaction costs (e.g. Burness and Quirk (1980b), Colby (1990), and McCann and Easter (1999)). These studies deal largely with issues different from the ones we consider, primarily pertaining to the optimal definition of a water right under a variety of conditions while we take the definition of a right as given. Parts of our model framework builds upon the insights in Bennett et al. (2000), which examines the optimal choice of interstate water compacts.

the efficiency of arbitrary rights allocations, allowing society to weigh users differently, and thus prioritize specific uses (producers). Using the model set-up outlined, we show that equivalence of arbitrary rights to socially optimal outcomes only holds under restrictive conditions, which are not frequently encountered in reality. Put another way, even if society wishes to assign different priority to different users, via assigning higher weight to certain users, many existing rights allocations cannot be supported by any valid set of priorities (weights), due to the magnitude of the divergence between water allocations implied by the rights ('water allocations')) and productivity of use ('productivity'). Clearly, allocative efficiency, requiring all producers to be weighted equally, is even less likely to be attained under these conditions. Another insight we are able to offer is this: any market-based mechanism of voluntary water transfer (e.g. trade) that takes the existing set of rights as given, while welfare improving, cannot lead to the optimal (or efficient) allocation of water. We emphasize that this aspect, of not attaining optimality in allocation, is an inherent feature of the mechanism of assigning water rights, and unrelated to other aspects such as transaction costs, which only serve to further reduce scope for reallocation.

The second question also relates to a point noted in Burness and Quirk (1980b), that shortages in a water delivery system can be artificially induced by institutions, an issue whose welfare implications we explore in a controlled setting using a simulation approach. We are particularly interested in two questions relevant for policy in an increasingly water-scarce world: (i) the degree to which trade can ameliorate (exogenous) water shortages, and how it relates to the correlation between productivity of use and the granted rights; and (ii) the extent to which transaction costs (conceptualized as ad valorum taxes on traded water) affect water trade, and how this relates to the said correlation. We are interested in this correlation precisely because many common rights systems are based on historical usage rather than on current productivity, and either lack mechanisms for productivity increasing transfers of water rights, or have restrictions imposed upon such transfers (Colby (1990); Gould (1989)). These are, in the words of Ciriacy-Wantrup (1956), the legal aspects that 'decisively influence' economic efficiency. In fact, the extent to which rights diverge from productivity is, in a stochastic sense, the extent of inefficiency induced as a result of legal aspects.

Our results indicate that in the face of scarcity, the degree to which trade can help is limited, and depends upon the initial legal allocation. With a lower correlation between productivity and rights, scarcity-induced increases in trade are relatively small (less than 3%), and are largely offset by the increase–resulting both from scarcity and transaction costs of trade–in the price of water (at 15%). Turning to the effects of transactions costs, we find them to exert a surprisingly moderate effect upon both trade and gains from trade (reduction of less than 20% with a tax as high as 40%), with only gains from trade being affected substantially by the correlation between rights and productivity. The key insights from our framework are this: when trade in water is allowed between producers whose marginal valuations of water are not widely different (e.g. farmers along a river), scarcity need not induce the anticipated large reallocation of water, and this is particularly true when the initial allocation bears little relation to productivity. In essence, rents created by water entitlements that do not correspond to productivity are by far the larger impediments to reallocation of water, compared to the transactions costs of trading water.

How to deal with existing water allocation mechanisms is an important question to answer when determining appropriate policy options to encourage more effective use of existing resources. The exercises we carry out demonstrate the limits of a market in recovering losses due to economic inefficiency from water entitlements that are not aligned with current productivity. Trade is always beneficial, but water rights lead to the accrual of economic rents to some users, with an associated social cost realized in the form of reductions in trade, primarily via the intensive margin. These results arise in a setting where social efficiency is equivalent to economic efficiency, indicating that in settings with a more expansive definition of social efficiency, e.g. including environmental quality, the effectiveness of the market is likely even further reduced. Accounting for these sorts of concerns require a more careful consideration of the many facets of incompleteness of property rights, not just their duration and transferability, including aspects regarding who is allowed to participate (e.g. what counts as a "beneficial use"). These facets of property rights are particularly relevant for the question of environmental flows, an aspect of significant importance in many semi-arid contexts, including the Western U.S. where they are currently incompletely accounted for (Grafton et al. (2011)).[6]

The rest of the paper proceeds as follows. Section 2 will present the basic model, using which two distinct frameworks of welfare will be considered: the socially optimal in section 3, and individual entitlements with trade in section 4. Section 5 will present simulations and relative welfare results, with a focus on how they relate to model parametrization. Section 6 presents a detailed discussion of the major results and key policy-related insights, while section 7 will conclude. Additional simulation-related details, mathematical details, and tables of simulation results are provided in the Appendices (A, B and C).

2 Modeling

We take a world with a single resource, which is the only factor necessary for production, abstracting away from important real world considerations such as input substitution and technical change. For the purposes of discussion we will focus on the case where the resource is water and production is agriculture, although with minor modifications this framework could be used to think about other contexts or resource scenarios. There are N producers, indexed by $i = \{1, 2, \ldots, N\}$, denoting their physical priority for the resource. We utilize a partial-equilibrium framework, where the final good has an exogenous price,

[6] Environmental flows have been recognized as a beneficial use even in many Western states. However, responsibility for these flows is more ad hoc in the United States, left to state governments, quasi-governmental agencies and private organizations. This is in contrast to Australia, where it is federally mandated that environmental water be provided for, and these supplies are provided via the water resource planning process and government purchases of water rights when adjustments are necessary (Grafton et al. (2011)). See Garrick et al. (2009) for a detailed comparative case-study of the Columbia River in Washington State and the Murray-Darling Basin of Australia.

for simplicity normalized to unity, rendering production equivalent to revenue. Each producer chooses his water withdrawals, and has a quadratic production function in water utilization,

$$y_i = f(u_i) = a_i + b_i u_i - c_i u_i^2, \tag{1}$$

with the following marginal product,

$$MP_{u_i} = \frac{\partial y_i}{\partial u_i} = b_i - 2c_i u_i. \tag{2}$$

Schoengold and Zilberman (2007) suggest this functional form is appropriate to describe behavior at the farm level, while Cobb-Douglas type functions, of which the exponential is the special case for a single input, are more appropriate for macroeconomic scenarios. We will have more to say about the specific issue of production functions later on, but we note that for the case of a hypothetical market with many producers considered in section 4 (and for the policy simulations in section 5), the suggestions of Schoengold and Zilberman (2007) are very pertinent: it is neither realistic nor interesting to use the exponential production for modeling or to conceptualize a market with many small individual producers. There is an initial water inflow level, $\overline{W}$, which, following e.g. Johnson et al. (1981), may be viewed as the amount available for use after accounting for any required in-stream flows and outflows. This assumption (partly) accounts for flows for non-economic goals such as environmental management, allowing us to use the sum of producer profits as our measure of aggregate welfare. Each producer has his own return flow rate for water withdrawals, $0 < \delta_i < 1$, thus $0 < (1 - \delta_i) < 1$ is producer i's consumptive use.

2.1 Free Withdrawls

The foregoing set up means that, when allowed to withdraw water freely from this river, i.e. treating the river as an open access source, each producer's available water, W_i^f, for $i \geq 2$, can be expressed by means of the following recursion,[7]

$$W_i^f = \max\left\{W_{i-1}^f - (1-\delta_{i-1})w_{i-1}, 0\right\} = \max\left\{\overline{W} - \sum_{j=1}^{i-1}(1-\delta_j)w_j, 0\right\}, \tag{3}$$

with $W_1^f := \overline{W}$. In the absence of any restrictions on water use, utilization, u_i, will equal withdrawals, w_i, ceasing when there is no more water available or when their marginal product of water utilization reaches 0:

$$u_i^f = w_i^f = \min\left\{W_i^f, \frac{b_i}{2c_i}\right\}, \ \forall i. \tag{4}$$

[7]We note that both our flow constraint equations, eq. (3) for the free access case and eq. (5) for the realized legal alloation, are written out recursively, instead of on an individual-by-individual basis, as in e.g. Weber (2001); Johnson et al. (1981).

This level is a reflection of the technological or productivity limit to each producer's water demand. In this case, no individual producer's decision depends on any δ_i, they are only relevant in that they determine the total level of water available in the system. An important feature of available water, W_i^f, from eq. (3) is worth noting. An increase in water available in the system, $\overline{W}$, or in any return flow, δ_i, would lead to increased water supply (and hence withdrawals), since increased return flows have essentially the same effect as an increase in $\overline{W}$.[8]

2.2 Legal allocation

Next, following the practice in the literature (e.g. Burness and Quirk (1979)), we add a perfectly enforced all-or-nothing right to withdrawal, $\gamma_{J(i)}$, where $J(i)$ is user i's legal priority. This notion of the legal status of rights, while clearly a simplification, is adopted in the interests of model tractability.[9] The addition of legal rights leads to two notions of 'space' in our framework. First a 'natural' ordering of users along a river in a linear fashion according to index i, and is standard in the previous literature (e.g. Weber (2001)); secondly the legal ordering, which can either be linked to i in some organized fashion or be arbitrary, depending upon the rights system modeled. More formally, if $i = \{1, 2, 3, 4, 5, \ldots, N\}$ is the natural ordering of users, then the legal ordering, $J(i)$, may be thought of as a one-to-one re-ordering of this sequence. For simplicity of notation, we will alternatively denote the legal ordering by j_i or as only j when the discussion is focused on the rights themselves. This distinction between the natural and legal order is sufficient for our purposes of investigating the welfare consequences of different relationships between rights and productivity.[10]

With the introduction of a legal right, a new consideration that arises is the potential for a gap between withdrawals and use of water. Legal priority exogenous with respect to i may be considered characteristic of many water allocation systems, including the Australian statutory rights system and the Western U.S. appropriative rights system, where water rights can be detached from land.[11] From the perspective of allocative efficiency two key facets of inflexible rights are: that they are often tied to a *specific* 'beneficial use', often with explicit or implicit barriers to changing that use, and they often have a 'use it or lose it' nature. The latter feature has been noted (e.g. Burness

[8]Changes in δ_i potentially have differential effect upon intermediate users, and have implications upon these users similar to environmental flows discussed in section 6.

[9]The definition of a "legal right (entitlement) to water" is widely varying and frequently complicated in practice. In the context of the Western U.S., it is often a right to withdraw a fixed amount of water, with the rules specifying 'seniority' having implications for for sharing under scarcity (Libecap (2010)). In the context of statutory rights in Australia, there can be as many as three distinct rights, with a base water entitlement, an administratively determined "water allocation" to the entitlement, varying from year-to-year, and a water license to use the allocated water (Grafton and Horne (2014)).

[10]We note that the need for two notions of space arise primarily due to our use of different allocation mechanisms with differing notions of space. Related studies (e.g. Weber (2001)) explore other aspects (e.g. in-stream flow benefits or third party externalities) that render location particularly important.

[11]A strictly proportional rights system requires setting $\gamma_{j_i} = s_i\overline{W}$, and if s_i is some measure of relative access to the water source in question (e.g. frontage on a river) then this can be interpreted as a form of a riparian system. For the proportional case, if there is any water at all available, all users will necessarily receive a positive allocation.

and Quirk (1980b)) for its tendency to encourage inefficient production choices, such as wasteful irrigation practices or the planting of low-profit margin crops. Such rights systems tend to arise in areas with abundance of water resources relative to demand in order to encourage economic development and water use, e.g. the context of appropriative rights in the U.S. (Leonard and Libecap (2016)) and statutory rights of the Murray-Darling Basin of Australia (Grafton and Horne (2014)). This is in contrast to riparian rights, which are linked to land and where all users have unlimited access to water that flows over or adjacent to their land, only subject to agreed upon modifications with other rights holders. From our perspective what is important is that riparian rights prioritize flexibility, while appropriative rights prioritize security.

The framework of welfare we seek to develop is intended to model the essential features of water allocation regimes which prioritize security to the exclusion of flexibility. We note that in flexible rights systems, e.g. a proportional rights regime, these problems related to allocative inefficiency are not as prominent. Firmly motivated by many real-world examples, and following earlier literature (e.g. Johnson et al. (1981), Burness and Quirk (1980b)), we are focused on basins or water systems which are over-allocated, and therefore assume that $\sum_{i=1}^{N}\left[(1-\delta_i)\,\gamma_{J(i)}\right] > \overline{W}$.[12]

Regardless of the precise structure of the legal rights, availability of water is now contingent upon legal rather than physical priority. In this legal ordering, the first producer's *realized* allocation is the simple and intuitive expression $\gamma_j^l = \min\left\{\overline{W}, \gamma_j\right\}$. For producer 2 onwards, the realized water allocation is now[13]

$$\gamma_{j_i}^l = \max\left\{\min\left\{\overline{W} - \sum_{h=1}^{j-1}\left[(1-\delta_h)\,\gamma_h^l\right], \gamma_j\right\}, 0\right\},\ j \geq 2. \tag{5}$$

Note the similarities between availability in eq. (3) and eq. (5), with availability determined by the withdrawals of upstream users in the former case and by the rights of users with higher legal priority in the latter. This leads to the following demand functions,

$$w_i^L = \gamma_{J(i)}^l, \tag{6a}$$

$$u_i^L = \min\left\{\frac{b_i}{2c_i}, w_i^L\right\}, \tag{6b}$$

[12]Common examples of hydrologically over-allocated basins are the Colorado River basin in the western United States (Christensen and Lettenmaier (2006)) and the Murray-Darling basin in Australia (Pittock and Finlayson (2011)). In our context, it is not necessary that the water be *legally* over-allocated. Rather, what is necessary is scarcity in the system i.e. at the current level of productivity, there is insufficient water for everyone's full use, formally represented as $\sum_{i=1}^{N}\left[(1-\delta_i)\,u(i)\right] > \overline{W}$. As will be seen from our pricing equation, Equation (19), the implied price in the decentralized case will be zero in a basin without scarcity or over-allocation.

[13]The expression for γ_j^l may be easier to understand by considering a specific producer, say producer (in legal order) 2 for simplicity. The component of consumptive use of the realized allocation, γ_1^l, for producer 1 is $(1-\delta_1)\,\gamma_1^l$, leading to the total available water in the system being $\left[\overline{W} - (1-\delta_1)\,\gamma_1^l\right] \geq 0$. Thus, the allocation for producer 2 can be at most $\min\left\{\overline{W} - (1-\delta_1)\,\gamma_1^l, \gamma_2\right\}$.

for producer i, $i = 1, \ldots, N$.

We can see from Equation (5) and Equation (6) that technology, legal structure and physical scarcity of the resource completely determine the behavior of the system. For model tractability any unused withdrawals (i.e. $w_i - u_i > 0$) are still subject to the loss δ_i. Thus, $(1-\delta_i)w_i$ is returned to the system, regardless of how much is utilized for productive purposes. This means that a gap between w_i and u_i causes reductions in water availability for all users downstream of i without any corresponding benefit to producer i, an added source of inefficiency. From this we can also see that producers can be separated by how scarcity affects the realization of their legal allocation. Denoting respectively by F and P the number of producers who receive their full legal complement and the number who receive a partial share, it is clearly the case that by assumption $F + P < N$. So, $\gamma_j^l = \gamma_j$, for $j = 1, \ldots, F$, $0 < \gamma_j^l < \gamma_j$, $j = F+1, \ldots, F+P$; evidently, $\gamma_j^l = 0$, $j = F+P+1, \ldots, N$.

3 The Social Planning Problem

We turn now to the social planning problem. For any scarce resource, achieving economic efficiency requires equalizing the marginal value of that resource, considered as a productive input, across active uses. That value, in turn, will be the market price of the resource in question, should a market exist, or a shadow value, if not. Value in this context has always included the relevant measurable values of market goods and services, while recent developments have spurred the inclusion of non-market goods and services as well, using a variety of techniques to provide monetary valuations for non-traded factors (Howe (1998)). We assume that the available water supply already accounts for such non-market externalities, to set aside these issues in order to focus on the welfare consequences of the water distribution from a producer's perspective.

There is a long history in utilitarian welfare economics of considering outcomes that are 'socially optimal', in the sense of maximizing the welfare of all individuals in society. In the context of our problem, this maximum does not have to represent the maximum level of profits that the system can achieve, because a social planner can favor some producers over others, through the social planning weights. Firmly in this tradition, we consider a benevolent utilitarian social planner, who assigns such a weight, ξ_i, to producer i. In our context, these weights may arise for a variety of reasons, including reflecting public policy decisions to prioritize certain users (Howe et al. (1982), Molle and Berkoff (2006)).

We denote the social objective function by $\mathcal{S} = \sum_{i=1}^{N} \xi_i \pi(y_i)$, where, following the discussion above, π is the profit function of an individual producer and $\{\xi_i\}$ is the set of social planning weights, with $\xi_i \geq 0, \forall i$ and $\sum \xi_i = 1$. It is evident that this form of a welfare function encompasses the setting where economic efficiency is being maximized by setting $\xi_i = \xi \ \forall i$, an objective that been used frequently in the prior literature.[14] The

[14]It should also be evident from the optimality conditions (eq. (9)) that our notion of socially optimal is weaker than allocative efficiency, since the planner can allocate, via larger weights to certain less productive

social planner will maximize welfare, $\mathcal{S}$, subject to water availability from eq. (3) and accounting for allocated-but-unused water. This leads to the following Lagrangian,[15]

$$\max_{u_i, w_i} \mathcal{L} = \left\{ \sum_{i=1}^{N} \left(\xi_i \left(a_i + b_i u_i - c_i u_i^2 \right) + \mu_{u_i} (w_i - u_i) \right) + \mu_w \left(\overline{W} - \sum_{i=1}^{N} (1 - \delta_i) w_i \right) \right\}, \tag{7}$$

with first order conditions

$$\frac{\partial \mathcal{L}}{\partial u_i} = \xi_i \text{MP}_i - \mu_{u_i} = \xi_i (b_i - 2c_i u_i) - \mu_{u_i} \leq 0 \tag{8a}$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \mu_{u_i} - (1 - \delta_i) \mu_w \leq 0. \tag{8b}$$

The unconstrained social planner will always choose $w_i = u_i$ because any un-utilized water has the potential for productive use downstream. Thus we can set all first order conditions for utilization equal to zero and combine them to obtain the following set of optimality conditions,

$$\begin{aligned} \mu_w &\leq \frac{\xi_1}{1 - \delta_1} \text{MP}_1 = \frac{\xi_1}{1 - \delta_1} (b_1 - 2c_1 w_1) \\ &\vdots \qquad\quad \vdots \qquad\quad \vdots \\ \mu_w &\leq \frac{\xi_N}{1 - \delta_N} \text{MP}_N = \frac{\xi_N}{1 - \delta_N} (b_N - 2c_N w_N), \end{aligned} \tag{9}$$

where equality holds for all i with $w_i > 0$. Combining these first order conditions, we arrive at $N - 1$ equations relating all producers to a single reference producer. Adding in the resource constraint leaves us with N equations in N unknowns. This a standard result, with the social planner choosing each level of water withdrawals, and therefore utilization, so that the relative marginal productivity of all active producers is the same, with the additional caveat that the social planner must determine which, if any, producers will not operate.[16]

To characterize the entire socially optimal allocation, we need to identify a reference producer to begin this relative allocation process and then adjust until the resource constraint is satisfied. For an exponential production function, any producer with a

producers, more water than would be indicated purely by efficiency considerations (efficient allocations for a simple case are presented in footnote 17). Consequently, we will refer to these allocations as "optimal", rather than as "efficient".

[15] Note that the welfare framework we use is different from the one used in the groundwater literature (see e.g. Appendix B of Schoengold and Zilberman (2007)) and that the rights we consider differ from the common groundwater rights framework (discussed in e.g. Burness and Quirk (1980b)). The optimization problem in eq. (7)-(9), for the case of equal weights, is similar to the one used in Weber (2001, §2) for answering a different question regarding joint surface water and pollution rights allocation.

[16] Only in cases where the planner is confronted with existing water rights (as in Johnson et al. (1981)) or instream-flow-values (as in Weber (2001)), is there scope for relative MPs not to be identical, due to the flow constraints also appearing in the problem. In such cases, unlike in ours, there is scope for third party externalities or location-specific prices.

positive social planning weight will end up with a positive allocation, so any producer can be the reference. For a quadratic production function, however, the social planner may wish to shut down some producers. We need to determine the set of producers with positive allocations, $\mathcal{W}^+$, and choose our reference producer from this set. For simplicity, and w.l.o.g., we take this reference producer to be the first, with allocation w_1^s. Then, for $i \neq 1$, we can combine all of the first order conditions and determine the following allocations,

$$w_i^s = \begin{cases} \frac{b_i}{2c_i} - \left[\frac{\xi_1(1-\delta_i)}{\xi_i(1-\delta_1)}\right]\frac{b_1}{2c_i} + \left[\frac{\xi_1(1-\delta_i)}{\xi_i(1-\delta_1)}\right]\frac{c_1}{c_i}w_1^s & \text{if } i \in \mathcal{W}^+ \\ 0 & \text{else.} \end{cases} \tag{10}$$

Using the resource constraint, we can determine w_1 to be

$$w_1^s = \frac{\overline{W} - \sum_{i\in\mathcal{W}^+}\left[(1-\delta_i)\left(\frac{b_i}{2c_i} - \frac{\xi_1(1-\delta_i)}{\xi_i(1-\delta_1)}\frac{b_1}{2c_i}\right)\right]}{\sum_{i\in\mathcal{W}^+}\left\{(1-\delta_i)\left[\frac{\xi_1(1-\delta_i)}{\xi_i(1-\delta_1)}\frac{c_1}{c_i}\right]\right\}}. \tag{11}$$

For each producer, there are two components of the social planner's water allocation: a first order productivity adjustment, where producers are rewarded with additional resources for having a large b_i relative to b_1, and a second order productivity adjustment, for having a small c_i relative to c_1. In each case, the term "relative" accounts for differences in return flow efficiency, social planning weights and any additional multiplicative factors we might add to the model (e.g. externalities or taxation). As a result, having identical values for parameters is not necessary for two producers to have equal allocations.

A few simple features of the socially optimal allocation are interesting are and worth mentioning. Consider the simplest case with equal social planning weights, identical return flows and second-order productivity parameter, i.e. $\xi_i = \xi, \delta_i = \delta$ and $c_i = c$, $\forall i$. In this case only the first order productivity term differs across users and the optimal allocation, seen in eq. (10) and eq. (11),[17] has two striking features: first, each producer's water allocation exceeds or falls below the reference producer depending only upon whether his productivity (parameter b_i) is larger or smaller than that of the reference producer; second, producers who do not satisfy a productivity cut-off do not receive any allocation–which also turns out to be the case for unequal weights. This latter feature is the key difference between the optimal allocation in the de-centralized (i.e. free withdrawal) case (see eq. (4)), in which every user receives and uses some amount of water until the supply of water is exhausted.[18]

[17] The allocations simplify to: $w_i^s = \frac{1}{2c}(b_i - b_1) + w_1^s$ and $w_1^s = \frac{\frac{\overline{W}}{(1-\delta)} - \sum_{i\in\mathcal{W}^+}\frac{1}{2c}(b_i - b_1)}{N^+}$, where $N^+ \leq N$ is the number of individuals receiving a positive allocation. The set $\mathcal{W}^+$ includes all producers i such that $b_i > b_1 - 2cw_1^s = \mathrm{MP}_1$.

[18] This necessarily implies the de-centralized outcome, based purely upon spatial ordering, is itself inefficient.

With these aspects in mind, we investigate the relationship between legal allocation of water rights, summarized in eq. (5) and eq. (6), and the socially optimal allocation, given by eq. (10) and eq. (11). Key to the social planning problem are the social planning weights, and our approach is to seek to understand whether, and under what conditions, a set of weights exist which will give rise to equivalent welfare under the two allocation mechanisms.[19] A surprising and unique–to our knowledge–result is that a clear relationship between these two forms of allocating water can indeed be established. This is our key result for this section, and is formalized next.

Proposition 1. *For any well-behaved (convex and twice differentiable) production function over the relevant domain, any arbitrary set of legal rights, $\{\hat{\gamma}_j\}$, such that $\sum_{i=1}^{N}(1-\delta_i)\hat{\gamma}_{j_i} > \overline{W}$, corresponds to a set of social planning weights, $\{\hat{\xi}_i\}$. This set of planning weights is unique if every marginal product evaluated at $\hat{\gamma}_{j_i}$ is non-zero.*

Proof. Start with any arbitrary allocation such that $\sum_{i=1}^{N}(1-\delta_i)\hat{\gamma}_{j_i} > \overline{W}$. We re-index rights around $j = P$, the last producer to receive their full water allocation, so that all realized rights sum to $\overline{W}$.[20] Without loss of generality, we will also assume that $\sum_{i=1}^{N}\hat{\xi}_i = 1$. The social planner desires a relative allocation of water that satisfies the optimality conditions in Equation (9). The social planner takes each producer's marginal product (MP) as given, and thus will evaluate it at the full level of utilization, $u_i = w_i = \tilde{\gamma}_{j_i}$, for the purposes of allocation. The only exceptions to this rule arise when accounting for producers who shut down. If $\tilde{\gamma}_{j_i} = 0$, then $\hat{\xi}_i = 0$, regardless of the value of MP_i. After excluding such producers, we are left with the set of active producers, $\mathcal{W}^+$, of cardinality N^+. For all members of this set we have that,

$$\frac{\hat{\xi}_k}{(1-\delta_k)}\text{MP}_k = \frac{\hat{\xi}_i}{(1-\delta_i)}\text{MP}_i \qquad \forall i,k \in \mathcal{W}^+. \tag{12}$$

A corner case arises if $\text{MP}_i = 0$, for any i. Then for all k where $\text{MP}_k \neq 0$, we take $\hat{\xi}_k = 0$, and for all i such that $\text{MP}_i = 0$ we may choose any set of values $\{\hat{\xi}_i\}$ which satisfy $\sum_{i\neq k}\hat{\xi}_i = 1$. In this case the solution clearly need not be unique.

The socially optimal outcome would be to allow only individuals who are sufficiently productive (i.e. those with marginal productivity greater than or equal to the shadow value of water) to draw water. In the de-centralized case, water is effectively treated as a common pool resource with the amount a producer can withdrawal being completely unrelated to their productivity. Clearly, a specific instrument would be necessary to compensate for this externality, an aspect we do not consider here.

[19] While the broad outlines of our approach bears some similarity to the celebrated result of Negishi (Negishi (1960)), our setting is very different. The points of similarity are related only to the broad ideas: using a weighted sum of individual welfare measures and in finding equivalence between two distinct forms of resource allocation via the weights. Consequently, the results of Negishi are not applicable to our case. Indeed, for our main production function, the quadratic, positive weights only exist under restrictive conditions, as shown in corollary 1.

[20] This is accomplished by setting, $\tilde{\gamma}_j = 0$ for all $j > F+P$, $\tilde{\gamma}_j = \hat{\gamma}_j$, for all $j \leq P$ and $\tilde{\gamma}_j = \tilde{\gamma}_{j-1}(1-\delta_{j-1})$ for $P < j \leq F+P$. The resulting set of rights $\{\tilde{\gamma}_j\}$ will trivially satisfy the resource constraint.

If there are no producers for whom $\text{MP}_i = 0$, then Equation (12) implies the following relationship between weights for any two producers i and k in $\mathcal{W}^+$,

$$\hat{\xi}_k = \hat{\xi}_i \left[\frac{\text{MP}_i}{\text{MP}_k} \frac{(1-\delta_k)}{(1-\delta_i)} \right], \tag{13}$$

which leads to the following expression for $\hat{\xi}_i$,

$$\sum_{k=1}^{N} \hat{\xi}_k = \sum_{k=1}^{N^+} \hat{\xi}_k = \sum_{k=1}^{N^+} \hat{\xi}_i \left[\frac{\text{MP}_i}{\text{MP}_k} \frac{(1-\delta_k)}{(1-\delta_i)} \right] = \hat{\xi}_i \left(\frac{\text{MP}_i}{(1-\delta_i)} \right) \left(\sum_{k=1}^{N^+} \frac{(1-\delta_k)}{\text{MP}_k} \right) = 1$$

$$\Rightarrow \hat{\xi}_i = \frac{(1-\delta_i)}{\text{MP}_i} \Big/ \sum_{k=1}^{N^+} \frac{(1-\delta_k)}{\text{MP}_k}. \tag{14}$$

Combining this with *eq.* (13) yields the following equation,

$$\hat{\xi}_k = \hat{\xi}_i \left[\frac{\text{MP}_i}{\text{MP}_k} \frac{(1-\delta_k)}{(1-\delta_i)} \right] = \frac{(1-\delta_k)}{\text{MP}_k} \Big/ \sum_{k=1}^{N^+} \frac{(1-\delta_k)}{\text{MP}_k}. \tag{15}$$

This is the same as *eq.* (14), and thus describes the entire set of social planning weights, $\{\hat{\xi}_k\}$. These values depend only on the values of δ_i and MP_i, which is itself dependent on the level of rights, $\tilde{\gamma}_j$.

To see that this set is unique, consider the following. To increase the value of any member, say $\hat{\xi}_1$, would require lowering the value of at least one other individual's weight, say $\hat{\xi}_2$, to ensure that the weights still sum to 1. However, this implies that the new set cannot be identical to the first, and thus must differ from the solution from eq. (15). □

This result is general, without the need to specify a specific production function within the proof. Key to this generality is the fact that the weight of any individual producer need not be positive. Negative weights would result from over-allocations, i.e. producers with legal allocations that imply negative marginal products. Those marginal units of water could otherwise be reallocated to a positive marginal product producer and increase overall welfare, so with strictly positive weights those allocations would not be possible. Next, we summarize some conditions on production functions that are sufficient to ensure that all weights are positive.

Corollary 1. *The following are sufficient conditions for a production function to yield a non-negative sequences of weights, $\{\hat{\xi}_i\}$, for an arbitrary rights allocation $\{\tilde{\gamma}_{j_i}\}$ as determined in Proposition 1:*

(i) the function has a positive marginal product for all possible values of water input, $0 \le w_i \le \overline{W}$ (no negative or zero marginal products); and

(ii) there exists a positive number $A < \infty$, such that $MP_i < A$ for all w_i s.t. $0 < w_i \le \overline{W}$ (no infinite marginal products, except at zero).

The exponential production function $f(w_i) = w_i^\alpha$ with $0 < \alpha < 1$ satisfies these conditions because its marginal product $MP_i = \alpha w_i^{\alpha-1}$ is positive for all $w_i > 0$, and is only infinite at $w_i = 0$ otherwise being bounded above by $A := \alpha \left(\gamma_j^+\right)^{\alpha-1}$, with γ_j^+ the minimum positive legal allocation observed. The quadratic production function we will use henceforth satisfies the second condition as long as $b_i < \infty$ and $c_i > -\infty$, but will fail the first condition in many cases. This can be overcome by restricting the possibilities for the sequence of legal rights such that $\tilde{\gamma}_{J(i)} < w_i^\star$, where $w_i^\star$ is the water value such that $\text{MP}_i = 0$. For many realistic producer-level production functions, allocations that do not satisfy condition (i) may be termed 'socially infeasible', since they cannot be replicated by a social planner with any feasible set of weights. To this extent, therefore, the results of Proposition 1 and Corollary 1 should be viewed as illustrative of the magnitude of allocative inefficiency resulting from water entitlement assignments disconnected from current productivity.

An important implication of Proposition 1 is this: regardless of intent, society indirectly determines the distribution of welfare gains through the manner in which it allocates water rights. The status quo of maintaining existing rights structures is equivalent to a transfer from those with allocations below their marginal productivity to those with allocations above their marginal productivity. Our framework helps highlight the connections between these two methods of allocating water, including their equivalence when Corollary 1 holds.

Historically-determined water rights distributions, particularly those determined when making productive use of under-utilized resources was the policy goal, invariably involve the previously noted trade-off between security and flexibility. In this context, significant alterations to resource conditions (e.g. from surplus to scarcity) and relative productivity ensure that deviations from initial conditions will tend to introduce, or increase, economic inefficiency. To the extent that water-related infrastructure-investment is shorter-lived relative to water rights tenure, as is true in many parts of the world where individual investments are under consideration, dynamic resource allocation efficiency may require the legal regime of water allocations to be flexible enough to account for significant changes in circumstances (a view articulated before, e.g. Grafton et al. (2000), Conning and Robinson (2007)).

Another insight offered by our results here pertain to the oft-discussed efficiency-equity trade-off in water allocation (Libecap (2010)). In many over-allocated basins, the assignment of water rights do not correspond to current productivity or marginal valuation of water. In this context, the lack of a socially feasible set of weights implies that a market-based re-allocation of water entitlements (e.g. a long-term lease of water/sale of water entitlements) is Pareto-improving, in addition to clearly being welfare improving. Thus under these circumstances, addressing efficiency need not involve a trade-off with regards to equity. Although derived in a very simplified context, this insight is nonetheless consistent with the scattered empirical evidence (Libecap (2010, pp. 19-21)) providing support to the thesis that entitlement reform in a developed country setting can often have only a minimal impact on key post-reform aspects of concern. Evidently a different set of circumstances, including definition of water entitlements, and additional dimensions of importance in a real-world setting, such as the process of facilitating a transfer of entitlements or other political economy-related issues, may either prevent some of these benefits from materializing or even lead to adverse distributional consequences.

It is interesting to compare the broad outlines of our results to those in Weber (2001). That study provides an equivalence result between the efficient allocation and the market equilibrium in a problem of allocating joint water and pollution rights. Our results, on the other hand, indicate that the equivalence between the efficient (or the socially optimal) and legal (water rights-based) allocation is possible only under rather restrictive conditions not often associated with many existing water entitlements. Consequently, the post-trade equilibrium with existing legal allocations is highly unlikely to be efficient, even using our weaker definition of efficiency, as illustrated with simulation examples in section 5.2 (see also the discussion in section 4.3).

To further explore the link to welfare outcomes, we turn to the converse question, what effect do social planning weights have on the optimal distribution of water use? In general, an arbitrary set of social planning weights need not correspond to a unique and valid allocation of water. A non-trivial example is the quadratic production function, where the social planner often needs some firms to not produce, which can lead the social planner to desire non-valid (i.e. negative) allocations. There is however, a clear and convincing result for exponential production functions, a proof of which is presented in Appendix C.

Corollary 2. *With an exponential production function in water use (and parameter $0 < \alpha < 1$), an arbitrary set of valid social planning weights, $\{\hat{\xi}_i\}$, always implies a unique set of legal rights, $\{\hat{\gamma}_j\}$, that achieves the socially optimal water allocation.*

In the exponential case the marginal product of every producer is always positive, which means that none of them will choose to shut down if they can get a positive amount of water and that without a rights system, the first user will withdrawal and use all of the available water. While this very behavior is undesirable for a market composed of individual producers, it does provide a powerful result. The combination of proposition 1 and corollary 2 means that for an exponential production function, legal rights and social planning allocations (via planning weights) are perfectly equivalent ways of allocating resources.

Having characterized the relationship between the legal and the socially optimal allocation, we turn to a de-centralized set-up, considering a market for the trading of water derived from the previously detailed legal entitlements. The socially optimal allocations, and the associated modeling framework developed, provide the foundation and context for the de-centralized case, which will be presented as generally as possible analytically and then explored empirically using simulations.

4 The Decentralized Problem with Trade

4.1 The Setup

We now turn to a static model of water trade between users with water rights established under a legal setting in which allocations of water do not directly correspond to current values of marginal product (or productivity of use), there is scarcity in the system, and permanent trades in water entitlements are effectively prohibited.[21] These settings can be observed under a variety of water rights systems, including those under the appropriative doctrine and the Australian water rights system under over-allocation. For our purposes, it suffices that water allocations satisfy the above-stated conditions. The question we are interested in answering relates to the degree to which trade in water can address the initial misallocation. In other words, does the level of divergence of historically-determined legal entitlements from those based upon current productivity determine the extent to which introducing trade in water enhances allocative efficiency in cases of resource overallocation? This is a question suitable to the setting of smaller-scale markets that are already operational, involving temporary trade in water allocations across similar users, under the legal conditions detailed.[22] Conditions similar to these govern many transactions in water markets in Australia (e.g. trading of annual water allocation among farmers, usually within-state (Grafton and Horne (2014)) and the Western U.S. (e.g. single-year sales of water between users, discussed in Brewer et al. (2006) and Libecap (2010)).

We envisage a world consisting of N producers with heterogeneous productivity and a water market with an effective water price, p_T, accounting for any taxes or externalities that may be present. The market setting we envisage is modeled on a few operational ones, comprised mostly of within sector trade in a single basin or legal jurisdiction, where there is not a wide dispersion of water valuation across users. Our market is conceived as operating thus: there is a single clearinghouse (administered by a regulator) where bids for sales/purchases of water are made, which lead to a single final price for a unit of water. This market is also assumed to be competitive, with no strategic interaction among participating producers. In this sense, our focus is on determining the endogenous price of water, abstracting away from more complex issues of how in reality such a price is likely to arise, and the informational requirements involved in the process of price determination. As a result, we do not pursue the more involved approaches investigated for price determination in specific markets pursued in e.g., Weber (2001), Wong and Eheart (1983).

In the presence of externalities, the adjustment to the price can be viewed as being

[21] When water rights are freely and permanently transferable, following an initial assignment of water rights there will be a re-allocation between users of these entitlements, involving more efficient users purchasing rights from less efficient ones, analogous to the case of pollution permits. In essence, the allocation of water rights in such a world merely creates rents for the fortunate holders of these rights, in so far as initial rights are not vested with those possessing the highest productivity of water use (or the highest value of marginal product). Consequently, a market for water will only arise in response to unforeseen temporary contingencies (droughts, productivity shocks, etc.), with any permanent change in economic (e.g. productivity) or resource (e.g. scarcity) conditions leading, via sales of rights, to a new, efficient holding of entitlements.

[22] Consequently, we do not consider two important types of water trade, between different sectors and across river basins. Trade across sectors results from large differences in marginal valuation, and often include political economy considerations beyond strict productivity that may be resisted by current holders of rights (Molle and Berkoff (2006); Olmstead (2010)), sometimes necessitating changes to existing water entitlements. Clearly, importing (or exporting) of water at the basin level is not directly relevant to our specific question. That said, our scenario of exogenous changes in water supply may be also be interpreted in light of unpaid for imports (uncompensated exports) of water.

due to two separate effects. The first is a "natural" level of externality, reflecting the physical costs and limitations of water trade.[23] The second is a pure ad valorum tax, a constant unit tax per unit of water traded, which can be used to adjust demand to account for factors such as basin scale externalities or to raise revenue to finance projects such as infrastructure maintenance/development or ecological restoration. Thus the price can be decomposed as

$$p_T = (1 + \tau + \epsilon)p_T^0, \tag{16}$$

where p_T^0 is the price absent any market distortions, τ is a tax or other transaction cost component and ϵ is a natural externality. Either component could be zero and it is possible that the τ could be negative, e.g. a subsidy designed to offset the cost of a particular natural externality. Such components are seen within existing pricing regimes, e.g. direct charges for water resources management in South Africa (Rep of South Africa (2015)) or subsidies to agriculture resulting in urban prices above cost of service delivery in Australia (Crase et al. (2015)).[24] If $\tau + \epsilon$ is non-zero, then there is a difference between the price paid by buyers of water, p_T, and the price received by sellers, p_T^0. For simplicity of exposition, we will assume that $\epsilon = 0$, since for the purposes of our exercise an externality is indistinguishable from a tax. The tax therefore may viewed as arising from the administrative cost of operating the market and maintaining public infrastructure, in essence a simplified version of market pricing as outlined by the most recent reform of the National Water Act in South Africa (Rep of South Africa (2015)). Either price, p_T or p_T^0, can be taken as exogenous and assumed to be 'market clearing,' i.e. equalizing the amounts of water sold and bought.

With trade each producer chooses u, w by optimizing post-trade profit,

$$\max_{u_i, w_{Ti}} \left\{ a_i + b_i u_i - c_i u_i^2 - \left[p_T^0 + I(w_{Ti} > 0)\tau p_T^0\right] w_{Ti} \right\}, \tag{17}$$

subject to the constraints

$$0 \leq u_i^T \leq w_i, \tag{17a}$$

$$w_i = \gamma_{J(i)}^l + w_{Ti}, \tag{17b}$$

$$w_{Ti} \geq -\gamma_{J(i)}^l, \tag{17c}$$

where $\gamma_{J(i)}^l$ (from eq. (5)) is the realized legal allocation for producer i with priority $J(i)$, with w_T^i his traded water.[25] We work with the convention that sales of water

[23] Not all externalities take this form, that is multiplicative and constant per unit of water used. As will be discussed later, water rights of the type we discuss turn out to be one such non-linear externality. These can only be compensated for with additional policy instruments beyond an ad valorum tax.

[24] A commonplace scenario, and one which we abstract away from in considering a single type of use, is that of heterogeneous treatment in pricing. This provides scope for more complex taxation schemes that may be used to target goals beyond economic efficiency (Sjödin et al. (2016)).

[25] For simplicity, and wherever no confusion can result, we refer to water derived from the legal entitlements as the 'legal allocation' of water. Clearly, all allocations, legal or socially optimal, pertain to withdrawal, since that is the basis upon which legal entitlements are usually defined.

enter with a negative sign and purchases with a positive sign. Thus, sellers have their costs reduced while buyers' have their costs increased. Note that profits from production are adjusted to reflect the revenue from (cost incurred on) sales (purchases) of water. The first constraint is the same activity constraint as in previous optimizations; the second defines withdrawals as rights plus the amount traded; the final constraint simply limits producer's sales to his legal allocation, although he may buy as much as can be productively used.

It is important to note that with trade, for any positive price p_T^0, $u_i = w_i$, i.e. no producer will want to simply 'let the water through.' This should be self-evident, since any (gross of return flow) unit of water unused for production can now be sold. For both buyers and sellers utilization will equal their rights adjusted accordingly for the quantity traded; thus, $u^T = \gamma_{J(i)}^l - w_{Ti}$ for sellers and $u^T = \gamma_{J(i)}^L + w_{Ti}$ for buyers.

4.2 Price Determination

Starting with the pre-trade allocations, quadratic production functions lead to linear marginal conditions, so we know that there will be price cutoffs which fully determine desired trading behavior. Denoting sales of water ($w_{Ti} \leq 0$) as w_{Si} and purchases ($w_{Ti} > 0$) as w_{Bi}, solving for each producer's trade quantities yields the following expression for traded quantities,

$$w_{Ti} = \begin{cases} w_{Si}^1 = \gamma_{J(i)}^l & \text{if } p_T > p_i^Q, \\ w_{Si}^2 = \left(\gamma_{J(i)}^l - \frac{b_i}{2c_i}\right) + \frac{p_T}{2c_i}\frac{1}{(1+\tau)} & \text{if } p_i^S \leq p_T < p_i^Q \text{ or } \gamma_{J(i)}^l > \dfrac{b_i}{2c_i} \\ 0 & \text{if } p_i^B < p_T < p_i^S \\ w_{Bi} = \left(\frac{b_i}{2c_i} - \gamma_{J(i)}^l\right) - \frac{p_T}{2c_i} & \text{if } p_T \leq p_i^B, \end{cases} \tag{18}$$

where $p_i^Q = b_i(1+\tau)$ is the shutdown price, $p_i^B = b_i - 2c_i\gamma_{J(i)}^l$ is the buying cutoff price and $p_i^S = \left(b_i - 2c_i\gamma_{J(i)}^l\right)(1+\tau)$ is the selling cutoff price. There is a strict ordering of these cutoff prices, $0 \leq p_i^B \leq p_i^S \leq p_i^Q$, with the first equality only occurring if $\tau = 0$, the second if user i has no water available under the non-market legal regime (i.e. $J(i) > P$), and $p_i^B = p_i^Q$ if, in addition, there are no taxes. A producer would only buy subsidized water if $p_i^B < 0$, while $p_i^S < 0$, implies that $\gamma_{J(i)}^l > \frac{b_i}{2c_i}$, and that producer i will at a minimum sell his excess water at any positive price.

These price conditions and how they relate to a producer's participation in the market are determined by the relationship between productivity and rights. Producers for whom $\gamma_{J(i)}^l \geq \frac{b_i}{2c_i}$ will always sell, reducing some of their production in favor of water sales, with the exact level determined by the price. Meanwhile, producers for whom $0 < \gamma_{J(i)}^l < \frac{b_i}{2c_i}$ can be either sellers or buyers. In particular, as evident from the expressions for w_{Si}^1 and w_{Si}^2, some users with a 'deficient' but positive legal allocation will sell if the price is high enough to displace their marginal production. Thus while sellers are heterogeneous, buyers are homogeneous: only users who have a 'deficit' will buy. As the discussion progresses it will be helpful to split producers into subgroups depending on their relationship with the market price, p_T, with a further sub-division into groups with 'excess' or 'deficient' allocation relative to productivity, i.e. depending upon whether $\gamma^l_{J(i)} \geq \frac{b_i}{2c_i}$ or $\gamma^l_{J(i)} < \frac{b_i}{2c_i}$, denoted respectively by '+' and '-'. This leads to the following groupings of producers, with $\mathcal{I}(.)$ an indicator function:[26]

- $A_i = \mathcal{I}\left(p_T \geq p_i^Q\right)$ (sellers who shut down)
- $B_i^+ = \mathcal{I}\left(p_T < p_i^Q\right)$ and $B_i^- = \mathcal{I}\left(p_i^S \leq p_T < p_i^Q\right)$ (normal sellers)
- $C_i = \mathcal{I}\left(p_i^B < p_T < p_i^S\right)$ (sellers priced-in to full production)
- $D_i = \mathcal{I}\left(p_T \leq p_i^B\right)$ (buyers).

These groupings relate to the extensive and intensive margins of trade, describing both which producers participate as buyers and sellers, and what quantity they choose to trade.

Developments thus far have been premised on a known market price, p_T, and an understanding of how such a price will arise in our market framework is the task taken up next. While the expression for the market price is relatively involved, and not amenable to comparative static analyses, it will turn out to have a relatively intuitive interpretation, particularly for certain simple cases.

Proposition 2. *For the case of the quadratic production function, the final market price is determined by the ratio of excess demand in the system to the total intensive margin of changes in water demand for all market participants.*

Proof. In equilibrium the quantities sold should equal the quantities bought:

$$
\begin{aligned}
0 &= \sum_S w_{Si} - \sum_B w_{Bi} \\
0 &= \sum_i \left(w^1_{Si} + w^2_{Si}\right) - \sum_i \left(w_{Bi}\right) \\
0 &= \sum_i \left[\gamma^l_{J(i)}\mathcal{I}(A_i) + \left(\gamma^l_{J(i)} - \frac{b_i}{2c_i} + \frac{p_T}{2c_i}\frac{1}{(1+\tau)}\right)\mathcal{I}(B_i)\right] \\
&\quad + \sum_i \left[(0)\,\mathcal{I}(C_i) - \left(\frac{b_i}{2c_i} - \gamma^l_{J(i)} - \frac{p_T}{2c_i}\right)\mathcal{I}(D_i)\right]
\end{aligned}
$$

$$
\Rightarrow p_T = \frac{\sum_{i=1}^{N}\left[\left(\frac{b_i}{2c_i}\right)\mathcal{I}(B_i \cup D_i)\right] - \sum_{i=1}^{N}\left[\gamma^l_{J(i)}\mathcal{I}(A_i \cup B_i \cup D_i)\right]}{\sum_i \left[\left(\frac{1}{2c_i}\frac{1}{(1+\tau)}\right)\mathcal{I}(B_i)\right] + \sum_i \left[\left(\frac{1}{2c_i}\right)\mathcal{I}(D_i)\right]} \tag{19}
$$

□

The price is determined jointly by all producers who participate in the market, all buyers and any sellers who sell any amount beyond their excess supply. The price depends upon total supply, $\overline{W}$, only indirectly, via the feasible legal allocation, $\gamma^l_{J(i)}$. Thus we

[26] Note that due to the effect of relative productivity in determining the cutoffs, C_i^+ and D_i^+ are both empty sets.

see that the legal water rights act as a non-linear transaction cost, as it is not only their magnitude but also their distribution that affects the price. The denominator is comprised of the sum of the second order productivity effects of the buyers and sellers, yielding a total measure of the intensive margin of changes in demand expected for a given set of market participants. Consequently, if water demand is more responsive (i.e. has a higher aggregate price elasticity of demand) then there is a higher willingness to trade for any price level, leading to lower water price than otherwise.

Identifying the effects of changes in system parameters is difficult for general cases, because they affect both the intensive and extensive margins of participation. However, we can reduce the parameter space to gain some intuition, before making some observations regarding the general case. For the case where $\delta_i = \delta$ and $c_i = c$, and there are only two types of producers, those with low-productivity/high-allocation and high-productivity/low-allocation, the expression for the price simplifies to the average marginal product of legal allocations multiplied by an adjustment factor that must be greater than one and represents the effects of taxation. Alternately, it can be viewed as the average of the respective participation cut-off prices (p_i^B and p_i^S) of buyers and sellers, weighted by their frequency and with sellers additionally discounted by the tax rate (see Appendix C for further details). In either case, we can clearly see that the price is linked to the distribution of legal rights, as that is a major factor in determining the initial state of relative productivity, and thus the willingness of different producers to participate in the market.

In the more general case however, we can only make a few observations to a first approximation. First, since legal allocations only appear explicitly in the numerator, it can be conjectured that the (post-tax) market price is decreasing in $\overline{W}$, since it is decreasing in $\gamma^l_{J(i)}$ and $\gamma^l_{J(i)}$ is increasing in $\overline{W}$. Similar is the case for increasing return flow, for reasons discussed in section 2.2. Finally, since taxes only appear in one term in the denominator, ceteris paribus we would conjecture that increased taxes to lead to increased market price. We stress that while intuitive and sensible, formally establishing the comparative statics of key aspects of the relationship between system parameters and price is not feasible in this setting. The endogeneity of price determination also implies that the system price depends only upon key system parameters and enables us to provide more realistic counter-factual policy simulations of interesting real-world issues.

4.3 Gains From Trade

We turn to computing the overall gains from trade, our chosen measure of welfare for the case with legal allocation of rights and trade. All actors within the system take the legal allocation of water as given, and total profits without trade are constant for a given allocation. The major difference between scenarios of legal allocations with trade and the social planning allocation is that the social planner can shut down relatively low productivity producers while the market can only provide incentives (payments) for shut down. Consequently, while individual profit for users who are subject to a shut down (or very low allocation) in a social planning allocation is higher under the market setting, overall welfare is lower. Another difference is that the social planner takes into account changes in the effective supply of water due to differences in return flow for potential buyers and sellers of water, while the market must pay for return flows to induce the same choices. Intuitively, one anticipates that, ceteris paribus, reduced supply of water (or reduced return flow, δ_i) will lead to a reduction in individual profits, and hence social welfare, while expanding the wedge between buyers and sellers (e.g. taxes) leads to lower trading, again with a corresponding reduction in welfare.

Denoting with superscript T and NT respectively objects with and without trade, by p_T^0 the pre-tax ('pure') price from eq. (19), and by $\mathcal{W}_l^+$ the set of all individuals who receive a positive legal allocation (i.e. $\mathcal{W}_l^+ := \{i;\, \gamma_{J(i)}^l > 0\}$), the gains from trade may be computed as (see Appendix C for details):

$$\begin{aligned}
\Phi &= \sum_{i=1}^{N} \left(\pi_i^T - \pi_i^{NT}\right) \\
&= \sum_{A_i^+} \left[p_T^0 \gamma_{J(i)}^l - \left(\frac{b_i}{2}\right)^2 \left(\frac{1}{c_i}\right) \right] + \sum_{A_i^-} \gamma_{J(i)}^l \left[p_T^0 - b_i + c_i \gamma_{J(i)}^l \right] \\
&\quad + \sum_{B_i^+} \left[p_T^0 w_{Si}^2 - \left(\frac{p_T^0}{2}\right)^2 \left(\frac{1}{c_i}\right) \right] + \sum_{B_i^- \times \mathcal{W}_l^+} \left[w_{Si}^2 \left(\frac{p_T^0}{2} - \frac{b_i}{2} + c_i \gamma_{J(i)}^l \right) \right] \\
&\quad + \sum_{D_i} \left[w_{Bi} \left(\frac{b_i}{2} - c_i \gamma_{J(i)}^l - \frac{p_T}{2} \right) \right].
\end{aligned} \tag{20}$$

Equation (20) indicates that the welfare effects on sellers divides them into six groups,[27] depending upon where their cut-off prices are and whether their natural demand, $\left(\frac{b_i}{2c_i}\right)$, is met pre-trade.

In any efficient market-based scenario, the final quantity is the total amount of value that can be captured, divided or otherwise bargained over. A noteworthy feature of the expression for gains from trade is that the legal allocations are, in most cases, sub-optimal, as was already indicated. The social optimum cannot be reached, even in the absence of a tax, since opportunity costs of resource use can only be realized with the spending of resources to incentivize and achieve water transfers from less productive to more productive individuals. In other words, this expression highlights the fact that legal rights serve as a mechanism that allocates resources to particular users who the social planner would grant a smaller allocation. Alternatively, these users receive an economic rent by virtue of their legal allocation.

From a policy perspective a key metric is how different facets of the model affect the gains from trade. These include the allocation of rights relative to productivity, the level of total water available ($\overline{W}$), return flows (δ) and taxation (τ). Basic economic considerations would seem to indicate that an exogenous reduction in supply (increase in $\overline{W}$ or decrease in δ), would increase scarcity in the system, and thus would tend to

[27]Sellers in $B_i^- \times \mathcal{W}_l^-$ have zero gains from trade because while they would sell, they have zero legal allocation. Sellers in C_i do not sell because the system price happens to fall in the gap between their buying and selling prices.

increase trade. However, this could potentially be counter-acted by the increased value of existing water, and thus the overall effect upon welfare depends on other systemic factors. An increase in taxes, on the other hand, causes a wedge between the price paid by buyers and the price received by sellers, and is unambiguously anticipated to lower trade, with a corresponding reduction in overall welfare.

We note that the expression for Φ is relatively involved, and does not permit an easy identification of the effects of changes in key system parameters upon welfare, ruling out comparative static analyses that are of interest. Nonetheless, as with the market price in section 4.2, a few tentative conjectures may be made. Given that the tax enters eq. (20) negatively, it may be conjectured that increase in tax leads to reduced gains. Much more ambiguous, even approximately, is the effect of exogenous changes in water supply, $\overline{W}$, which manifests via the γ^l-terms in eq. (20); since these terms appear with both signs, it is not analytically possible to sign the effect of changes in $\overline{W}$. This inability to easily sign the effects of changes in the supply of water on gains from trade is worth noting, particularly since it may appear at a first glance to be counter intuitive. However, supply changes affect both the extensive and intensive margins of trade, making overall effects difficult to predict a priori.[28]

We next turn briefly to understanding a few 'corner' cases, including the conditions under which gains from trade never arise. Briefly, it should be evident from eq. (18) that whenever water rights correspond identically to productivity (i.e. $\gamma^l_{J(i)} = \frac{b_i}{2c_i}, \forall i$), only users who are have zero availability, and a high productivity, will want to buy. Even when the legal ordering corresponds exactly to the productivity ordering, it is possible that a producer with no legal allocation will have a high enough marginal product to lead to a positive price and the displacement of existing production. In a world with many heterogeneous users and any legal allocation, it is likely always the case that gains from trade are non-zero.

5 Policy Simulations

It is important to emphasize that our simulations are intended to serve as a vehicle to explore the structure and mechanisms underlying a water market in many realistic settings. More precisely, the key objectives of our simulations are two-fold: first, to investigate the effect of the relationship between productivity and legal rights upon gains from trade; and second, to understand how the gains from trade and the system price are affected by key system parameters, including exogenous water shortages. Our approach to simulation follows a path similar to prior studies (e.g. Wong and Eheart (1983)), in that we consider a hypothetical market based upon parameters derived from an empirical setting. In addition, understanding the effects of key system parameters,

[28] To see this, note that exogenous reductions in $\overline{W}$ clearly reduces the number of individuals receiving legal allocations (setting $\gamma^l_{J(i)} = 0$ for some individuals). This is likely to affect both the extensive margin, with previous sellers having positive rights allocations now becoming buyers with reduced (often to zero) allocations, and the intensive margin, with price changes affecting the magnitude of trades from producers with unchanged allocations. Together, these affect the scale of both Π^T and Π^{NT} in ways that will alter the scale of Φ.

such as tax, overall supply, and return flows provides policy-relevant insight. We first outline a few details regarding the simulation framework before proceeding to analyze specific cases.

5.1 Simulation strategy and Details

Two aspects of the simulation are worth discussing briefly, the first being the simulation strategy allowing us to obtain draws from independent marginal distributions ('marginals') for productivity and rights, with varying degrees of (positive) correlation, and the second being choices of key parameters.

What is required, in terms of the relationship between productivity, b_i, and rights, $\gamma_{J(i)}$, is the following; that higher values of productivity are associated with higher values of γ, and vice-versa, where the marginals of b and γ are positive, with specified mean and variance. The choice of a distribution with positive marginals, fixed parameters of joint distribution, and varying positive association is a challenging one, and one we deal with using a copula approach, whose details are presented in Appendix A. The essence of this approach is easily summarized. The use of a copula, in our case the Gumbel-Hougaard (GH) copula, enables one, simply by varying the values of a parameter of the copula, denoted θ, to obtain draws from the required joint distribution (between productivity and water rights), F, without altering the parameters of the respective marginals. The choice of the *Gumbel-Hougaard* copula was motivated by the requirement of accommodating a widely varying range of correlations as well as being simple enough to facilitate simulation. In essence, varying values of θ will enable obtaining any desired (Kendall) correlation between 0 and 1. This flexibility is a key reason for the popularity of the copula in statistical modeling of dependence in a wide variety of fields and applications, including increasingly in econometrics (Fan and Patton (2014)).

We briefly detail our choice of key parameters next. The key parameters in our analysis include total system supply ('supply'), $\overline{W}$, return flow parameter, δ, and the productivity parameters, $\{b, c\}$, which play a key role in determining the optimality of any particular allocation. In the interest of simplicity, we only vary the first order productivity parameter, b, keeping c fixed across users (at 0.0014). Our choice for b varied across users, with a mean value of 3 and a maximum of 6. These parameter choices are based upon crop-specific empirical estimates in Moore et al. (1994), and are in units of water, ensuring that our measure of gains from trade is also in water units. Water rights, γ, were chosen to have a mean value of 1500, with a maximum of 3000. N and $\overline{W}$ were chosen at respectively 100 and 40713, to ensure that $\overline{W} < \sum_{i}^{N} (1-\delta)\, u_i(b_i, c_i)$ and that $\overline{W} < \sum_{i}^{N} (1-\delta)\, \gamma_{J(i)}$, reflecting the circumstances that are of policy interest.

Based upon existing literature about the general efficiency of irrigation technologies, the return flow coefficient, δ, was chosen to be identical for all producers, at 0.2, the median achieved level in Howell (2003). Differences in return flow rates provide an additional margin for supply changes, with trades potentially causing changes in total available supply in the system, depending on the relative difference between return flows for the buyer and seller. This is noted in Ward and Pulido-Velazquez (2008), who report that under common river basin conditions, irrigation conservation subsidies actually reduce return flows and negatively affect aquifer recharge. The inclusion of this additional source of producer heterogeneity requires specifying how return flows are related to productivity, and is left for future research. Key parameter values and distributions chosen are also summarized in table 4 in Appendix B.

5.2 Comparison of Water Allocations

We turn first to understanding the differences in water allocations between the three regimes considered: free access, social planner, and legal (with and without trade). For the first two cases, producer productivities are held fixed, while in any setting with a legal allocation, productivity and legal allowances are drawn jointly, as described above. In particular, we will focus on illustrative cases of moderately low ($\rho_\tau = 0.2$) and high correlation ($\rho_\tau = 0.825$), and note that the insights obtained for other correlations are similar to the one discussed here. Our distributional parameters for productivity and legal rights limit the variance in all cases, ensuring that producers do not differ very widely in their water demand, consistent with our discussion in section 4.1 and footnote 22. This is by design, otherwise a small subset of producers are likely to dominate production, leading to many of the water trades involving sales to them. Finally, for all our results presented, the social planner weights each individual equally, ensuring that the weights can play no special part in the allocation.[29]

Figure 1 compares the social planning allocation to the free access (decentralized) case.[30] In the free-access scenario, producers who are beyond the cut-off level of physical availability lie along the x-axis, while in the social planning case producers who are below a minimum productivity threshold fall along the y-axis, reflecting rationing by physical availability in the decentralized world and by marginal productivity in the planning case. In each case, for a given distribution of productivity the exact level of the cut-off between getting some water and getting none is dependent only on system parameters, $\overline{W}$ and δ.

With this distinction in mind we move to Figure 2, which presents allocation scenarios, respectively without and with trade, for cases where the correlation is low (fig. 2a and fig. 2b) and high (fig. 2c and fig. 2d). As before, legal and socially optimal allocations are 0 along the $x-$ and $y-$axes respectively. It is evident that cases without trade are relatively similar, regardless of correlation, with significant subsets of producers falling into two categories: those who are over-allocated from a social perspective and those who are not active. The average level of allocations is determined by productivity, with increased correlation merely reducing the spread of the resulting legal allocations around that level.

[29] Our choice of equal weighting was motivated by many factors, including their predominance in prior economic analysis of the efficiency of water rights and the interpretation of economic efficiency that results. In addition, for virtually all our empirical allocations, it will turn out that the conditions of corollary 1 are violated for at least one producer; as a result there is no equivalent set of planning weights that can be applied.

[30] Allocations above the 45° (red) line represent 'over-allocations' from the planner's perspective, while those below represent an 'under-allocation', an interpretation that is consistent across the graphs.

Trade, on the other hand, compresses allocations for all levels of correlation, resulting in shifts in water use from producers with positive legal allocations to previously non-active producers. Every producer is active when there is trade, which means that the market price is low relative to the marginal productivity of every producer at zero. In effect, the market outcome may be viewed as adding a hybrid rationing mechanism, with an initial rights distribution followed by a second stage, with trades that partially account for relative productivity. We can see that the resulting allocations are clustered less tightly in cases with lower correlations, indicating that the ability of the market to correct for allocative inefficiency is limited. Additionally, producers who are allocated more water than they would use to produce even without a market are compensated for their fortunate initial allocation. This rent payment necessarily leads to fewer trades, transfers which the social planner would make for free. Put another way, trades that would benefit society are not made as a result of the incentives of individual producers and the arbitrary (from the perspective of productivity) mechanism of allocating water rights.

To summarize, the three allocative mechanisms have clear differences. The free access allocations are highly inefficient, with some high productivity users receiving no water and most who do being over-allocated. With a legal allocation mechanism and no trade, a similar allocation results, with essentially arbitrary (in either a physical or productivity ordering) producers receiving the benefit of a generous legal allowance. The no-trade cases result in a tighter cluster around the average level of productivity as the correlation between rights and productivity increases, which in turn leads to similarly clustered set of allocations with trade. Increased correlation between productivity and rights reduces, without completely eliminating, any initial allocative inefficiencies. In many real cases of interest a producer's productivity is only moderately related to his water allocations, so the degree to which trade in water can correct for the overall level of allocative inefficiency appears to be limited.

Clearly, to the extent that incentives to invest in water-related infrastructure and efficient storage are an important factors related to the rights allocations, the deviation from optimal found here may be viewed as an upper limit on the degree of allocative inefficiency. Nonetheless, the magnitude of deviation from an efficient outcome we find is likely indicative of the degree to which existing water allocation mechanisms are sub-optimal, a finding consistent with that of much of the economics literature on water allocation mechanisms. What is new here is the illustration of the degree to which, in an empirically-motivated setting, trade in water is able to mitigate the resulting inefficiency, as well as an illustration of how it varies with the relationship between productivity and the initial allocations.

5.3 Gains from Trade and the Relationship between Productivity and Rights

We turn now to examining how the correlation between productivity and legal rights, ρ_τ, affects welfare. Two aspects of our system are worth reiterating. The first is what we

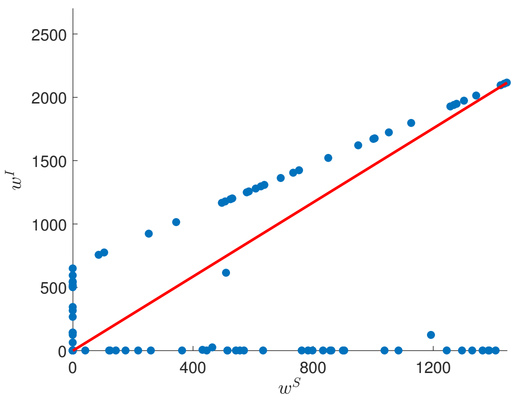

Figure 1: Socially optimal (w^S) and Individually optimal (w^I) allocation.

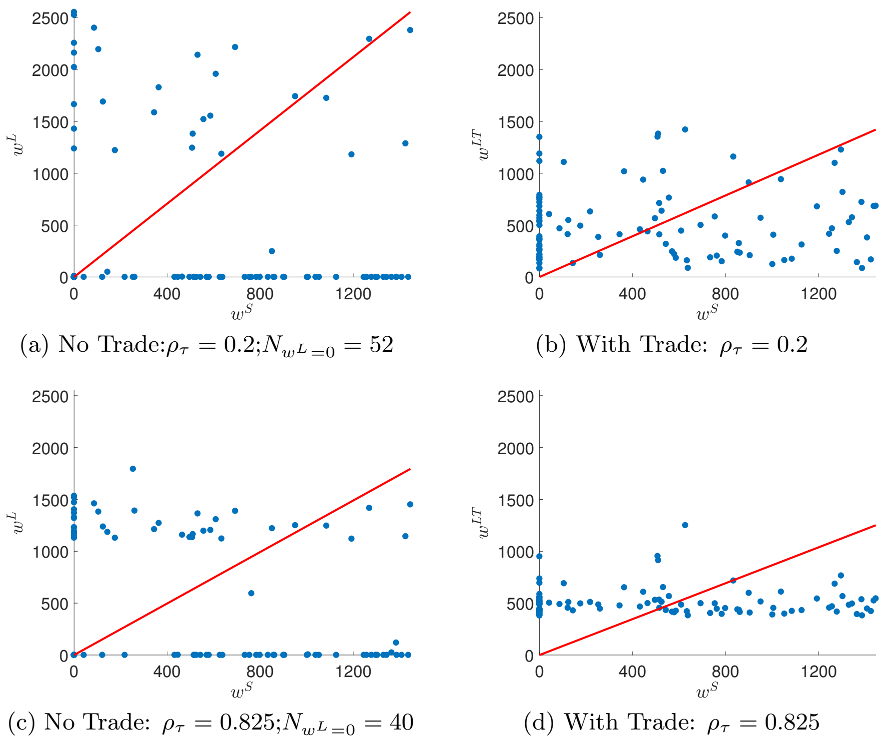

Figure 2: Socially optimal versus legal allocation (w^L,w^{LT}) for different levels of ρ_τ .

ρ_τ	Φ	**Trade ratio**	$N_{w=\gamma}$	$N_{w>0}$	p_T	N_S	N_B
0.05	8.472	0.846	26	47	2.173	27	73
0.1	8.337	0.846	26	47	2.123	27	73
0.2	8.024	0.843	27	48	2.02	27	72
0.35	7.405	0.834	29	50	1.848	28	71
0.5	6.645	0.811	32	52	1.659	30	69
0.75	4.942	0.741	36	57	1.287	37	63
0.825	4.428	0.717	39	60	1.158	40	60
0.9	3.965	0.694	42	62	1.02	42	58

Notes: Φ is in water units, with a scale of 10^4. $N_{w=\gamma}$ is the number of users who receive their full legal allocation, while $N_{w>0}$ is the total number of individuals who receive any positive amount of water. 'Trade ratio' is the ratio of water traded to the total available in the system, while N_S and N_B are, respectively, the number of sellers and buyers.
ρ_τ is (Kendall's) correlation coefficient between productivity and rights.

Table 1: Welfare and the correlation between productivity and rights

(loosely) term the 'natural demand' for water, the technology-based component of demand that is unaffected by prices, and the second is the 'market clearing' function of the price. It is the interactions between these two aspects that determine system outcomes. To set the stage for a discussion of the simulation results, it is worth summarizing our a priori expectation regarding the relationship between productivity and rights. Based upon basic economic considerations, one anticipates that reduced correlation between productivity and legal rights, resulting in an increased disparity between desired and allocated quantities, leads to a higher likelihood of trading and to larger gains from trade (for a given level of supply). The key aspects worth investigating are the degree to which trade is affected by changes in this correlation in a setting with realistic parameters; the channels through which these effects come about; and the degree to which they are dependent upon key parameters. This is the task undertaken here.

The results of our simulations are presented in Table 1, providing calculated values for key aspects of the system, including the market price, p_T, the trade ratio, and gains from trade. We look at a wide range of correlation values, from close to uncorrelated ($\rho_\tau = 0.05$) to very highly correlated ($\rho_\tau = 0.9$). Simulation results indicate, as anticipated, that gains from trade are decreasing with increasing correlation, albeit rather slowly. This is not accompanied, however, by a proportional reduction in the traded quantities, evident from the reduction in the 'Trade Ratio', from 84% to 69%.The high trade ratio even with high ρ_τ is explained by two factors. First, higher correlation does not completely eliminate mis-match between these factors, so the base levels of trade–for any legal allocation– are clearly non-zero for most allocations. Second, as already noted in section 4.3, even in the unlikely case of every producer's allocation being identical with productivity, scarcity and the very nature of legal allocations imply that there is scope for trade. This agrees with previous empirical findings, that higher values of production were associated with fewer trades (e.g. Brookshire et al. (2004)), since in our model a higher correlation between rights and productivity will (ceteris paribus) lead to an increase in the aggregate value of production. Gains from trade are reduced due to an increase in the individual legal allocation (see columns labelled $N_{w>0}$ and $N_{w=\gamma}$), and decreases in the price of water, which monotonically falls with increased correlation. Clearly, with a mean allocation of about 1500 and close to no correlation with productivity, fewer ($N_{w=\gamma} = 26$) individuals receive their full legal allocation of water. With the same mean, as allocations are tied more closely with productivity, the number of individuals who are receive a positive allocation ($N_{w>0}$) increases from 47, reaching a high of 62. To understand the pattern of trade, it is important to grasp the fact that the market price is endogenously determined; thus, reduced price with increasing ρ_τ is indicative of a reduction in aggregate excess demand for water. With this in mind, it is interesting to note that the number of sellers (N_S) is increasing with ρ_τ, implying that the average amount sold is falling. The converse is true of the number of buyers, which falls monotonically from 73 to 58. In fact, all producers engage in trade, for all-but-a-few levels of correlation, indicating that when there are two margins of trade, the quantity traded and the number of individuals trading need not be directly related, an aspect we will also explore in section 5.4.

To summarize the results of table 1, we find that reduced correlation between productivity and legal rights lead to increased gains from trade, reflected in the increased market price of water; the amount of water traded is less affected. For the particular parameterization considered, in fact, the gains from trade almost double when correlations fall from a very high 0.825 to a very low value of 0.1. While the broad direction of our results accord with economic intuition, we are able to quantify the gains from trade for a range of dependence between productivity and rights. That said, it is pertinent at this stage to again emphasize that trade merely reduces the welfare loss associated with mis-allocation of water. Many factors, including return flow payments, the nature of legal allocation (as a binding entitlement), and to a lesser degree taxation, prevent the post-trade allocation from achieving the socially optimal obtained in section 3, an aspect to which we return in section 6.

5.4 Trade and Key system parameters

Next we evaluate the relationship between gains from trade and key system parameters, in particular, supply, $\overline{W}$, and taxes, τ. Welfare comparisons with varying supply, however, must be made keeping in mind scale effects on the gains from trade.[31] This is also a reason for choosing the more plausible scenarios of changes in $\overline{W}$ (see also footnote 32). Consequently, it is somewhat difficult state a priori expectations regarding the effects of changes in supply upon the system. That said, one anticipates trade to increase with reduced supply; the effects of gains from trade, however, may depend upon many systemic

[31] Some remarks may be made regarding scale-effects, extending those in footnote 28. Note that a reduction (increase) in $\overline{W}$, for a fixed ρ_τ and legal allocation, clearly reduces (increases) the scale of π^{NT}. The degree to which π^{T} is affected depends upon two facets of the initial allocation: ρ_τ and the degree to which the basin is over-allocated. While it is highly unlikely that π^{T} would increase with a lower supply, to the extent that the initial allocation is distorted, and the basin is over-allocated, $\Phi^{\overline{W}} > \Phi^{\epsilon\overline{W}}$, for $\epsilon \in [0, 1)$.

factors, including the degree to which allocations diverge from productivity. The case of a tax is similarly ambiguous, with the effect upon traded quantities straightforward (an increase will reduce demand); effects upon other key system post-tax outcomes (price, trade and welfare gains) depend upon its effect on the extensive margin. We pursue further simulations to answer precisely these questions.

Parameter values	Φ	**Trade ratio**	$N_{w=\gamma}$	$N_{w>0}$	p_T	N_S	N_B	p_0
Level of tax, τ								
0	4.740	0.739	39	60	1.116	40	60	1.116
0.1	**4.428**	**0.717**	39	**60**	**1.158**	**40**	**60**	**1.053**
0.2	4.157	0.697	39	60	1.196	40	60	0.997
0.4	3.712	0.663	39	60	1.260	40	60	0.900
Changes in Supply ($\overline{W}$)								
default	**4.428**	**0.717**	**39**	**60**	**1.158**	**40**	**60**	**1.088**
20% higher	4.484	0.621	47	68	0.869	48	52	0.881
20% lower	4.100	0.810	31	52	1.443	32	68	1.343

Notes: Φ, 'Trade ratio', p_T, N_S, N_B, $N_{w=\gamma}$, and $N_{w>0}$ are all defined in table 1. All simulations pertain to the case of $\rho_\tau = 0.825$. Rows in **bold** refer to the base case for $\rho_\tau = 0.825$, from table 1.

Table 2: Welfare and key system parameters-High correlation

Parameter values	Φ	**Trade ratio**	$N_{w=\gamma}$	$N_{w>0}$	p_T	N_S	N_B	p_T^0
Level of tax, τ								
0	8.666	0.875	27	48	1.969	28	72	1.969
0.1	**8.024**	**0.843**	**27**	**48**	**2.02**	**27**	**72**	**1.836**
0.2	7.483	0.816	27	48	2.062	26	72	1.718
0.4	6.625	0.771	27	48	2.133	26	72	1.524
Changes in Supply($\overline{W}$)								
default	**8.024**	**0.843**	**27**	**48**	**2.020**	**27**	**72**	**1.971**
20% higher	8.892	0.798	33	54	1.733	31	68	1.092
20% lower	6.674	0.862	22	43	2.304	23	72	3.279

Notes: All simulations pertain to the case of $\rho_\tau = 0.2$. Other details are similar to those in table 2.

Table 3: Welfare and key system parameters-Low correlation

5.4.1 Taxation

We consider four different tax scenarios: no tax, and tax rates of 10, 20 and 40%, with the default scenario corresponding to a 10% tax rate, which is not far from the average rate of transaction costs of 6% found by Colby (1990). We also recall that taxes introduce a wedge between the buying and selling price, indicating that sellers are (ceteris paribus) less sensitive to taxes than buyers. We begin our analysis with the case of highly correlated allocation and productivity, with $\rho_\tau = 0.825$, results for which are presented in table 2. It is evident that gains from trade, as anticipated, fall with increasing taxation. Starting from the default value, eliminating the tax leads to a 3.1% increase in the trade ratio and an 7% increase in the gains from trade, while with an increase to 20%, the trade ratio decreases by 2.8% and the gains by 6.1%. Increase the tax to 40% leads to reductions in the gains from trade and the trade ratio, albeit at a slightly larger proportional effect than the increase from no tax to 10% tax (at 10.9% and 4.7% respectively).

Turning to the key metric of the market, post-tax price, we note that as anticipated, there is an increase with the tax, although surprisingly lower than the full amount of the tax, e.g. raising the tax from 0 to 40% leads to a price increase of only 13%. Clearly, this is a result of a substantial (19%) reduction in the system price (see column labelled p_T^0), and results from the following dynamic: increases in taxes lead to reduced underlying demand for water, and thus to a reduction in traded quantities. This leads to a reduction in the system price, p_T^0, thereby leading to reduced supply. In addition, for the parametrization considered here, the number of buyers and sellers is unaffected by the taxation, possibly due to the highly correlated relationship between legal allocation and productivity.

We turn next to the more policy relevant case of low correlation, results for which are presented in table 3. Relative changes in the trade ratio and welfare from eliminating the tax (beginning from the default) are only slightly larger than for the case of high correlation, at about 3.8% and 8%, while an increase to 20% leads to slightly larger reductions in gains and trade ratio (at 6.7% and 3.2% respectively). Further increase in the tax leads to proportionally larger reductions, and slightly larger than for the high-correlation case (at 11.5% and 5.5% respectively). Similar to the case of high correlation, the number of buyers is unchanged; the number of sellers, however, is reduced by 2.

Post-tax price increases are slightly lower than for the high ρ_τ-case, with an 8% increase in system price resulting from a 40% tax, with only a very slightly higher 23% reduction in the system price. The moderate increase in system price, compared to the high correlation case, is clearly driven by the higher underlying demand for water. Overall, while it appears that the (proportionate) effects of higher taxes are slightly larger at lower levels of correlation, the number of individuals trading, and the level of trade, are, as anticipated, far larger. In addition, the effects on trade of taxes appears slightly stronger, primarily driven by the extensive margin (with the number of sellers reduced by 2, as the tax increases from 0% to 40%).

To summarize, irrespective of the degree of mis-match between demand and supply of water, increases in taxes reduce gains from trade, with larger proportionate reductions seen in the case with lower correlation, and with only a moderate reduction ($< 15\%$) for the presented tax scenarios. Put another way, of the two margins for trade, taxation affects largely the intensive margin, with extensive margin effects being small and manifesting only in the case of low correlation. In light of this, the moderate reductions in trade resulting from the taxation scenarios considered here is not very surprising.

5.4.2 Changes in Supply

Next we consider the case of reduced supply of water in the system; in particular, we consider two cases, the first corresponding to a 20% increase in $\overline{W}$ and the second, and more policy-relevant one, corresponding to a 20% reduction.[32] Data from two recent droughts suggest that such a drop in water availability is not particularly extreme, and greater reductions in fact have been observed in many cases of practical importance. The Murray-Darling Basin of Australia experienced a drought from 2000-10, which led to inflows that were roughly 50% below the long term average for the ten years leading up to 2000 (Kirby et al. (2012)). Similarly, the 2007-09 drought in California resulted in runoff that was at least 36% below the hundred-year average for two major valleys (Christian-Smith et al. (2011)).

We again start with the case of high correlation (table 2). Turning first to the case of increase in $\overline{W}$, we find a moderate reduction in trade, at about 13%, and a larger drop in the market price (of about 25%). In accord with prior intuition, the number of sellers increase and the number of buyers decrease, both sizeably (by 8 each). These dynamics, evidently, are driven by an increase in the 'excess supply' of tradeable water. Interestingly, for the case of low ρ_τ (table 3), the fall in price and trade are both lower, at 14% and 5% respectively, and so too are changes in the number of buyers and sellers (at 4 each). Evidently, with a much larger mis-match between supply and demand, trade and price levels only fall moderately. To summarize, increase in supply leads to larger trade response in the high correlation case, with the extensive margin significantly contracting, consistent with basic economic intuition .

The more interesting case is that of reduced supply. For the case of high ρ_τ, this results in trade rising by about 13%, and price rising by a substantial 25%. The number of sellers fall, and the number of buyers rise, by a similar magnitude (8 each). The dynamic is easily explained: an across-the-board reduction in everyone's supply unambiguously increases the demand for water. For the chosen distribution of productivity, this implies that price rises sufficiently to increase the effective supply of tradeable water. It is important to note that this last feature is not a given: it depends upon key system parameters, including upon ρ_τ and productivity.

This aspect is more easily understood in the low ρ_τ caseIn this scenario, increase in trade is much lower, at only 2.3%, driven by a smaller increase in system price (at 14%). In essence, there is an asymmetric change in the extensive margin, with fewer sellers (by 3) but the same number of buyers. The latter aspect is interesting due to the number of individuals receiving water being reduced ($N_{w>0}$ is lower by 5). These rather surprising findings are driven largely by the previously stated counteracting effects of a shortage: the market price of water is higher but so is the production value of every unit of water. For a few sellers, clearly, the latter effect outweighs the former. Meanwhile the resulting price increase drives out a few buyers who may otherwise have been in the market. The effects upon gains from trade and consistent with the remarks in footnote 31 and footnote 28: reduction (increase) in $\overline{W}$ leads to lowered (increased) Φ, with a proportionately larger

[32] A word on the magnitudes of change in $\overline{W}$ chosen is in order here. Gains from trade, from eq. (20), are non-linear and scale-dependent, and our choice of parametrization is driven by an interest in keeping the resulting model sensible. Thus, large changes in $\overline{W}$, taking us beyond the limits of our model, are difficult to plausibly evaluate. Nonetheless, we note that our key insights are not very dependent upon the precise magnitude of change chosen. For instance, when changes of 10% were applied instead, the resulting insights were not significantly different than the ones discussed here (tables available upon request).

reduction (higher increase) at higher ρ_τ.

Our findings regarding changes in supply may be summarized as follows: for any correlation between productivity and legal allocation, reduction in $\overline{W}$ leads to more trade in water, with proportionately smaller increases for a scenario with large mis-match between demand and supply to begin with. However, there is very little increase, or an actual reduction, in the gains from trade, indicating that taking into account that the extensive margin is important when considering the implications of managing scarcity via voluntary market mechanisms. Comparing changes in supply to changes in taxes, we find sizeable and asymmetric effects on the extensive margin of trading in the former that were not observed with taxes, even with relatively large changes.

6 Discussion

At this juncture, it will be useful to summarize our simulation results, and discuss their connections with, and implications for, water policy. Instruments for water policy can be loosely separated into three groups: those that address system-wide issues, those that define user's interests and those that manage the impacts and consequences of use (Young (2014)). Our market falls primarily into the second category, leaving aside the details of issues such as system operations and third-party externalities to focus on the fungibility of any existing and future entitlements.

Overall, our simulation results illustrate a few important points, for a given distribution of water entitlements and productivity. First, welfare in cases where water rights do not bear a strong relationship to current productivity–true in many parts of the world but particularly so in the Western U.S. (Howe (1998); Tarlock (2001))–can be significantly enhanced when trade is introduced. Our contribution was to quantify this well-known benefit for the case of an empirically-grounded market with endogenous water price. In such a market, the magnitude of welfare increases are indeed sizeable for all possible correlations between rights and productivity. Naturally, these gains are higher with lower correlations, with the market serving to partially correct for this mis-match. Welfare effects are frequently assessed using an economy-wide framework, often a computable general equilibrium model (e.g. Diao et al. (2005), Berrittella et al. (2007)). This approach is well-suited to addressing the effects on factors that cross sectors, and trade across regions or countries, tending to forego micro-level realism and intuition in favor of aggregate tractability. Importantly, such analyses take existing institutions and legal allocations as given. Our analysis, on the other hand, was explicitly intended to evaluate and quantify this latter aspect, the relationship between productivity and institutionally determined allocations, and the resulting implications for trade and welfare.

A second point pertains to the additional impetus to trade, and the gains from trade, resulting from exogenous shortages of water. Results from section 5.4 indicate that the effects of exogenous water shortages upon trade is marginal, when compared to those induced by overallocation of water, with the difference being more pronounced when the water allocation process is less correlated with productivity (i.e. when the degree of misallocation is larger). Thus, when market participants share similar valuation of water–ruling out trades between e.g. highly subsidized users and those who face the full (market or shadow) price–there are scenarios under which trade is unlikely to lead to large re-allocations of water. Consequently, when productivity and water entitlements are only moderately related, as in many real world contexts, these entitlements merely serve to increase the size of the rent that many users can extract, limiting the scale of reallocation possible via trade. This indicates that one of the major drivers of trade in water is the very process of allocating water entitlements, in particular the overallocation and misallocation of water. Put differently, our results indicate that the shortage induced by institutions, in this case legal entitlements, can overwhelm that of even moderate-sized droughts. This point is reinforced by our results in section 5.3 (and Appendix B), which indicate that, in an overallocated basin, trade is of surprisingly sizeable importance, irrespective of the degree of correlation or overallocation. It is important to note that these insights are not dependent upon the magnitudes of water shortages chosen, within the limits outlined in footnote 32.

A related point arises from the results shown in Appendix B. There it is also observed that when the water system is more over-allocated than for the simulations in Table 1-Table 3 (i.e. with the same productivity but with larger mean legal allocation), then traded quantities do not vary much for different correlations between rights and legal allocations. In other words, while the benefits from trade follow similar patterns to that in Table 1, traded quantities change very little with this correlation. This illustrates yet another point not always evident: the magnitudes of traded quantities and welfare increases need not be directly related, as they are influenced differently by the relationship between productivity and rights.

Finally, our findings regarding the degree to which taxation affects trade in this setting are somewhat unexpected. Increases in taxes resulted in a decreasing system price, limiting the increase in post-tax prices and substantially moderating the effects on the share of water traded and welfare. Very few sellers were forced out of participation in the market due to the spread between the pre- and post-tax prices, even for relatively large tax increases. This indicates that modest taxation, particularly for beneficial use within the water system, e.g. conveyance maintenance or the mitigation of externalities, would be a desired policy when existing prices are below the average marginal product of use. However, total environmental externalities are not always as modest relative to current water prices, and that is a case that our simulations do not address.

Clearly, there are many important issues involved in managing water systems that have not been considered in our analysis. An important aspect of risk sharing is related to dynamics: multi-period models allowing for expanded trading options, including leases or permanent sales that cannot be distinguished in a static context, will be needed to analyze important aspects. Water rights structures in reality are likely more complicated than considered here,[33] adding significant complications to market implementation and analysis of the efficiency of existing allocations, minimally in the form of additional

[33] For instance, rights of different seniority would likely have different prices, with the expectation that older rights should have higher prices (Colby et al. (1993)). Alternatively, there may exist multiple levels of allocation, an example being the case where rights are controlled by irrigation districts, who then have autonomous control over allocations to their members.

transaction costs and uncertainty (Podolak and Doyle (2014)). We have also left aside third-party use externalities, e.g. water quality. Quality is particular has been discussed conceptually in the context of appropriative water rights (Fitzgerald (2012)), but to our knowledge empirical work on how they relate to water rights is absent. This is a key area for future research, since water quality can affect productivity in an agricultural setting, introducing more potential for feedback effects and problems that merely assigning rights may not easily correct. It is also important to note that while our analysis has been focused on economic efficiency, it is possible to expand the view of the quadratic production function to that of a more general benefit function that considers other outcomes, in monetary or utility terms, including the value of the environment. In-stream benefits, analyzed in a market setting in Murphy et al. (2009), also could be accounted for to work towards a more holistic analysis of the welfare issues presented in this paper.

Further issues could include social impacts – e.g. the need for different production or utility functions for small and large farmers, or different social planning weights due to factors such as the human right to water or the desire to support subsistence farming. It is not a given that markets will have positive distributional consequences, with Hearne and Easter (1997) showing that poor farmers had been negatively impacted by trades in some regions of Chile. To evaluate a wider set of benefits and allow them all to vary independently would at a minimum require a careful and nuanced analysis and potentially a model richer along relevant dimensions.

Finally, while we do not explicitly account for environmental flows, a few tentative remarks may be made as regards how it affects, and is affected by, key aspects we model. To the extent that environmental flow restrictions do not differentially affect users at intermediate locations, as our analysis assumes, their effect will likely be a reduction in overall supply. Naturally in the more interesting case when they do affect availability at intermediate locations, so long as their marginal values can be quantified (e.g. via having a user that is willing to purchase water), environmental use may be treated as another use(r) of water with a junior priority, subject to its being recognized as a 'beneficial use'.[34] In such a case, naturally, location will be important, leading to a market with many, potentially location-specific, prices, similar to the case of water quality considered in Weber (2001).

7 Conclusions

In summary, our analysis is able to disentangle and quantify the relative importance of two distinct aspects related to water entitlements that drive trade in water: overallocation and misallocation. It is the combination of overallocation and restrictions upon water entitlement transfers, which creates the conditions for our market for water. Misallocation alone need not lead to a functioning market, while overallocation always does in our setting, a feature arising as a consequence of the inability of the market to account

[34]As noted in Grafton et al. (2011), Australia and South Africa explicitly account for this in national level legislation. Further, in Australia, the U.S., and Chile rights can be purchased on existing markets for environmental uses.

for social inefficiency. Misallocation creates resource rents, which hinders the optimal use of water and provides additional incentives for trade in overallocated basins.

The intensive margin of trade is affected by both aspects, and to a lesser degree by transaction costs, while the extensive margin largely relates to the assignment of water entitlements. Exogenous shortages interact with these two aspects, resulting in a reduction in trade as misallocation increases, which is driven largely by the increased size of rents. In essence, when water is highly misallocated, as is often observed in practice, and when short-term water trading is already established, then droughts may only lead to very modest increases in trade. Recovering the socially efficient allocation would require outright transfers of allocations or user specific prices, in order to incentivize socially desirable but individually sub-optimal changes in water use.

An important consideration that we highlight is the key role of the completeness of property rights regime in managing water resources. These institutional frameworks determine aspects of social efficiency, equivalent to allocative efficiency for producers in our simulation explorations, that are not priced by the market, and the resulting rents stem from deviations of water allocation from those dictated by current economic productivity, adjusted for social preferences. The presence of additional unpriced factors, such as environmental quality, seems likely to increase the magnitude of both the rents and the resulting deviations from the socially preferred allocation.

Large-scale changes to property rights regimes can be challenging, generating political and legal issues and often being contentious, however our analysis indicates that incremental reforms may not always be completely successful in efficiently dealing with prolonged scarcity. For many settings with the characteristics considered here (scarcity, misallocation, and incomplete property rights), at least moderate reforms of the institutions, including property rights regimes, governing water may be necessary to ensure long-term sustainability and optimality of water use. In addition, recent reforms of property rights regimes have also often addressed the increasingly important question of environmental flows (e.g. Australia), indicating that addressing multiple facets of the incompleteness of property rights together may be politically and legally feasible. In any case, reform of any facet of property rights exerts a substantial effect on water use, thereby bringing the question of environmental flows into sharper relief.

Our model is relatively simple compared to most real-life scenarios, abstracting away from key issues such as environmental impacts and the complexity of water rights structure. Many features present in water systems, such as systemic aversion and resistance to change, multiple levels of seniority, additional distribution structures for legally granted water (e.g. irrigation districts), and legal limitations on transfers of rights,[35] complicate allocation mechanisms and their resulting welfare outcomes. Because of issues such as these, often dependent on local laws, a simple market as we outline is not likely to be sufficient to untangle existing regulations and institutional structures for water allocation (Podolak and Doyle (2014)). Given the importance of these types of externalities and institutional complications to real-world outcomes, a fruitful future line of research may

[35] Colby (1990) discusses a key example of third-party impacts leading to the legal blocking of a trade, *C*ITY & COUNTY OF DENVER BD. OF W. C. v. FULTON IRR. D. CO., 506 P.2d 144 (1972).

include the development of richer and more holistic analyses of the welfare issues raised here.

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Appendix A Simulation Details

We summarize a few details regarding the mechanics of the simulation.

A.1 Copula

Denoting by B_i and $\Gamma_{J(i)}$ the specified (known) distributions of b_i and $\gamma_{J(i)}$, respectively, of individual i, we seek a form for a joint distribution, denoted F_i, in cases where we wish the dependence between B and Γ to be specified using a parameter, called θ. This implies that the choice of B and Γ are distinct form the choice of the parameter θ specifying the degree of dependence. We always assume that B_i and $\Gamma_{J(i)}$ are independent across individuals.[36] To understand how this may be achieved, consider the following: assuming that B_i and $\Gamma_{J(i)}$ are both continuous, evidently $u_1 := B^{-1}(b_i)$ and $u_2 := \Gamma^{-1}(\gamma_{J(i)})$ are

[36] Note that depending upon the choice of parameter values for $\gamma_{J(i)}$, the constraint we wish to be met, $\sum_{i}^{K+1} \left(1-\delta_{J(i)}\right)\gamma_{J(i)} > \overline{W}$, for some $K < N-1$, will usually hold. Imposing this constraint explicitly will not, therefore, be required.

independently uniformly distributed. Next, given B_i, Γ_i, θ and F as above, a copula is a (distribution) function $C : [0,1]^2 \to [0,1]$ such that

$$F(b,\gamma) = C(B(b), \Gamma(\gamma); \theta).$$

To understand this equation, read from left to right i.e. if (b,γ) is a draw from the (unknown, so far) joint distribution F, then $v_1 = B(b)$ and $v_2 = \Gamma(\gamma)$ are two uniform random variables such that, for a known value of the parameter θ, $C(v_1, v_2; \theta)$ has the same distribution as F. Alternatively, if $U = (v_1, v_2) \sim C(.;\theta)$, then it is the case that $\left(B^{-1}(v_1), \Gamma^{-1}(v_2)\right) \sim F$. The latter expression, in fact, is the basis of our simulation strategy.

To make this discussion more concrete, if, as in our case, B_i and Γ_i are assumed to be β-distributed, and we wish them to have a (kendall) correlation of 0.5, and we choose the *Gumbel-Hougaard* copula C^{GH}, then a random draw from the unknown joint distribution F may be obtained by simulating random draws from $C^{GH}\left(u, v; \theta = \overline{\theta}\right)$ and inverting the relevant beta distributions. Clearly, varying the values of θ, one is able to obtain draws from the joint distribution F without altering the parameters of the marginal distributions B and Γ, a major advantage of this method. The choice of the *Gumbel-Hougaard* copula was motivated by the requirement of accommodating a widely varying range of correlations as well as being simple enough to facilitate simulation. The Gumbel-Hougaard copula may be written as,

$$C^{GH}\left(u_1, u_2; \theta\right) = \exp\left[-\left(\left(-\ln u_1\right)^{\theta} + \left(-\ln u_2\right)^{\theta}\right)^{\frac{1}{\theta}}\right], \theta \in [1, \infty).$$

A few key properties of this copula are worth mentioning. Kendall's rank correlation coefficient for this copula is $\rho_\tau = \dfrac{\theta - 1}{\theta}$. From this expression, it is evident that $\theta \to \infty$ corresponds to the case of perfect (positive) dependence, while $\theta \to 1$ corresponds to the case of independence (for which cases ρ_τ takes the value of 1 and 0, respectively). This copula belong to the family of so-called "Archimedean copulae", and has many interesting properties for which the readers are referred to standard sources: e.g. Nelsen (2007); Frees and Valdez (1998).

A.2 Price determination

A few points regarding p_T defined in eq. (19) are worth noting. First, the condition that there be at least one buyer and one seller implies that $\min_i\{p_i^B\} < p_T < \max_i\{p_i^Q\}$. Second, considering the equation for p_T as a fixed point equation and denoting the RHS as $f\left(p_T; \boldsymbol{\theta}\right)$, with $\boldsymbol{\theta}$ a vector of parameters, it is interesting to note that $f\left(p_T; \boldsymbol{\theta}\right)$ cannot be proved to be either continuous or monotonic. As a result, it is not clear that standard results regarding existence and uniqueness of a solution to the fixed point problem $p_T = f\left(p_T; \boldsymbol{\theta}\right)$ hold.[37] To ensure that our price is not a local solution to the

[37]In other words, iterative methods of solving for p_T may or may not easily work, and may depend upon parameter configurations $\boldsymbol{\theta}$. Equivalently, the intermediate value theorem does not hold for the problem

fixed point problem, we choose a random starting point between $\min_i\{p_i^B\}$ and $\max_i\{p_i^Q\}$ and check if the solution differs when one of the two end-points are chosen. For all cases considered here, we were able to confirm that the algorithm converged to the identical fixed point irrespective of which starting point was chosen.

A.3 Random draws from the Gumbel-Hougaard Copula

The procedure for simulating random variables from this distribution is somewhat non-trivial, and is detailed next.

Simulation Algorithm:

Step 1: Generate a rv X with from a stable distribution, $\mathrm{St}\left(\frac{1}{\theta}, 1, \gamma, 0\right)$, with $\gamma = \cos\left(\frac{\Pi/2}{\theta}\right)^{\theta}$.[38]

Step 2: Generate two independent $U(0,1)$ rv's v_1, v_2

Step 3: Set $u_i = G\left(-\frac{\ln v_i}{X}\right)$, where $G(t) = \exp\left(-t^{\frac{1}{\theta}}\right)$, $i = 1, 2$. u_1, u_2 are now samples from C^{GH}

Step 4: Set $b = B^{-1}(u_1)$ and $\gamma = \Gamma^{-1}(u_2)$. (b, γ) are now draws from the unknown joint distribution F.[39]

We note that an alternative, and simpler, approach to Step 1 may be developed, following Trivedi and Zimmer (2007, §A.1.3):

Step (1a) Draw a rv η from $U(o, \pi)$

Step (1b) Draw a rv ω from exponential distribution with mean 1

Step (1c) set $\alpha = \frac{1}{\theta}$, generate

$$z = \frac{\sin((1-\alpha)\eta)\,[\sin(\alpha\eta)]^{\frac{\alpha}{1-\alpha}}}{\sin(\eta)^{\frac{1}{1-\alpha}}}$$

Step (1d) Set $X = \left(\frac{z}{\omega}\right)^{\frac{1-\alpha}{\alpha}}$

Step(1e) X is now a draw from a positive stable distribution $PS(\alpha, 1)$.

	mean	range	marginal distributions
b	3 ($\bar{b}$)	(0.009,6)	scaled β(1,1)
γ	1500 ($\bar{\gamma}$)	(0,3000)	scaled β(1,1)
Default values			
		Tax rate, τ	0.1
		Total supply, $\overline{W}$	4.07×10^4
		Number of producers, N	100
		Return flow rate, δ	0.2
		Second order productivity parameter, c	0.0014

Table 4: Parameters and distribution choices

Parameter values	Φ	Trade ratio	$N_{w=\gamma}$	$N_{w>0}$	p_T	N_S	N_B	p_0
Level of tax, τ								
0	6.284	0.904	25	46	1.116	26	74	1.116
0.1	**5.905**	**0.886**	**25**	**46**	**1.143**	**26**	**74**	**1.039**
0.2	5.581	0.871	25	46	1.167	26	74	0.973
0.4	5.057	0.846	25	46	1.206	26	74	0.861
Changes in Supply ($\overline{W}$)								
default	**5.905**	**0.886**	**25**	**46**	**1.143**	**26**	**74**	**1.039**
20% higher	6.444	0.838	30	51	0.855	31	69	0.777
20% lower	5.189	0.940	20	41	1.428	21	79	1.298

Notes: Identical to table 2 but for a slightly more over-allocated scenario ($\bar{\gamma} = 2000$).

Table 5: Welfare and key system parameters-High correlation (more over-allocated)

Parameter values	Φ	Trade ratio	$N_{w=\gamma}$	$N_{w>0}$	p_T	N_S	N_B	p_T^0
Level of tax, τ								
0	9.531	0.928	22	42	1.969	22	78	1.969
0.1	**8.851**	**0.901**	**22**	**42**	**2.009**	**22**	**78**	**1.826**
0.2	8.279	0.877	22	42	2.044	22	78	1.703
0.4	7.373	0.838	22	42	2.101	22	78	1.501
Changes in Supply($\overline{W}$)								
default	**8.851**	**0.901**	**22**	**42**	**2.009**	**22**	**78**	**1.826**
20% higher	9.844	0.861	26	47	1.726	27	73	1.569
20% lower	8.233	0.984	17	38	2.295	19	77	2.086

Notes: Identical to table 3 but for a slightly more over-allocated scenario ($\bar{\gamma} = 2000$).

Table 6: Welfare and key system parameters-Low correlation (more over-allocated)

Appendix B Additional Tables

Appendix C Proofs

Corollary 2

Proof. The exponential function $f(u_i) = u_i^{\alpha_i}$, has marginal product $\mathrm{MP}_{u_i} = \alpha_i u_i^{\alpha_i - 1} = \alpha_i w_i^{\alpha_i - 1}$. From *eq.* (9) we can see that the social planner will attempt to equate the marginal products of each producer, weighted by $\frac{\xi_i}{(1-\delta_i)}$. Because this marginal product is always positive, the social planner will only choose $w_i = 0$ if $\xi_i = 0$, except in the case that every allocation is zero, i.e. $\overline{W} = 0$. So, $\mathcal{W}^+ := \{i; \xi_i > 0\}$

Without loss of generality we can take the first producer from the set to be the reference for the solution and combine *eq.* (9) with the resource constraint to results in the following non-linear equation in w_1,

$$h(w_1) = \overline{W}_0 - w_1 - \sum_{k \neq 1} \left(\frac{\alpha_1 (1 - \delta_k)}{\alpha_k (1 - \delta_1)} \frac{\xi_1}{\xi_k} \right)^{\frac{1}{\alpha_k - 1}} w_1^{\frac{\alpha_1 - 1}{\alpha_k - 1}} = 0. \tag{21}$$

We know this equation has a unique positive solution, from the intermediate value theorem $\left(\frac{\partial h}{\partial w_1} < 0,\, h(0) > 0 \text{ and } h(W_0 < 0)\right)$. With the non-arbitrary restriction that the social planning weights sum to 1, this allocation is unique.[40] This allocation satisfies the marginal product form of eq. (9) and the resource constraint and thus must be the same allocation as in proposition 1. ☐

C.1 Prices in a Reduced Parameter Space

Let us assume that $\delta_i = \delta$ and $c_i = c$ for all $i = 1, \ldots, N$ and that there are two types of producers, N_H of type high and N_L of type low, such that $b_H > b_L$ and $\gamma^H < \gamma^L$. With these simplifying assumptions, all low types are sellers and all high type are buyers, so

$p_T - f(p_T; \boldsymbol{\theta}) = 0$–in the absence of a proof that the function $g(p_T) = p_T - f(p_T; \boldsymbol{\theta})$ is continuous and/or monotonic–, indicating that existence and uniqueness via that approach is equally uncertain.

[38] A stable distribution is a four parameter family of probability distributions which includes many of the well known distributions–normal and inverse gaussian–as special cases. Except for these special cases, the pdf and cdf of this family cannot be written out explicitly, but its characteristic function can be. Simulations in Matlab used the "STBL" user written routine, found at https://www.mathworks.com/matlabcentral/fileexchange/37514-stbl--alpha-stable-distributions-for-matlab. See also http://math.bu.edu/people/mveillet/html/alphastablepub.html for a description of these functions.

[39] In our case, the inversion is only possible numerically, since the inverse CDF of the Beta distribution cannot be explicitly written out.

[40] The proof of this fact is the same as the proof regarding the social planning weights at the end of Proposition 1.

the expression for the price becomes,

$$p_T = \frac{\sum_{i=1}^{N}\left[\left(\frac{b_i}{2c_i}\right)\mathcal{I}(B_i \cup D_i)\right] - \sum_{i=1}^{N}\left[\gamma^{l}_{J(i)}\mathcal{I}(A_i \cup B_i \cup D_i)\right]}{\sum_i\left[\left(\frac{1}{2c_i}\frac{1}{(1+\tau)}\right)\mathcal{I}(B_i)\right] + \sum_i\left[\left(\frac{1}{2c_i}\right)\mathcal{I}(D_i)\right]}$$

$$p_T = \frac{\sum_{N_L}\frac{b_L}{2c} + \sum_{N_H}\frac{b_H}{2c} - \sum_{N_L}\gamma^L - \sum_{N_H}\gamma^H}{\sum_{N_L}\frac{1}{2c(1+\tau)} + \sum_{N_H}\frac{1}{2c}}$$

$$p_T = \frac{N_L\frac{b_L}{2c} + N_H\frac{b_H}{2c} - N_L\gamma^L - N_H\gamma^H}{\frac{N_L}{2c(1+\tau)} + \frac{N_H}{2c}}$$

$$p_T = \frac{N_L b_L + N_H b_H - N_L 2c\gamma^L - N_H 2c\gamma^H}{\frac{N_L}{1+\tau} + N_H}$$

$$p_T = \frac{N_L\left(b_L - 2c\gamma_L\right) + N_H\left(b_H - 2c\gamma^H\right)}{\frac{N_L + N_H(1+\tau)}{(1+\tau)}}$$

$$p_T = \left(\frac{N_L\left(b_L - 2c\gamma_L\right) + N_H\left(b_H - 2c\gamma^H\right)}{N_L + N_H}\right)\left(\frac{(N_L+N_H)(1+\tau)}{N_L + N_H(1+\tau)}\right) \tag{22}$$

$$p_T = \frac{\frac{N_L}{1+\tau}p_L^S + N_H p_H^B}{\frac{N_L}{1+\tau} + N_H} \tag{23}$$

The final two expressions are all equivalent, but each yields slightly different intuition for how the price is determined. In eq. (22), the price is the average marginal product, multiplied by a term dependent only upon the tax rate and the relative size of each group.[41] In this case we can clearly see how a positive tax leads to a non-linear distortion to the market price of water.

In eq. (23), we can see that the final price can also be viewed as the weighted average of the cut-off prices for sellers and buyers to engage in their respective market activities. The weighting consists of the number of sellers, discounted by the tax rate, and the number of buyers. Here we can clearly see again, that which producers buy and sell is deeply intertwined with the resulting market price.

[41] The aberrant case where the average marginal product is negative, which would yield a negative price, is averted due to our assumptions in the primary text, namely that $\sum \frac{b_i}{2c} > \overline{W}$. So we are necessarily assuming that water is scare relative to total demand within the basin.

C.2 Gains from Trade

$$
\begin{aligned}
\Phi &= \sum_{i=1}^{N} \left(\pi_i^T - \pi_i^{NT}\right) \\
&= \sum_{i=1}^{N} \left[b_i(u_i^T - u_i^{NT}) - c_i\left(\left(u_i^T\right)^2 - \left(u_i^{NT}\right)^2\right) - p_T w_{Bi} + p_T^0 w_{Si}\right] \\
&= \sum_{i=1}^{N} \left[(u_i^T - u_i^{NT})\left(b_i - c_i(u_i^T + u_i^{NT})\right) - p_T w_{Bi} + p_T^0 w_{Si}\right] \\
&= \sum_{A_i^+} \left[\frac{b_i}{2c_i}\left(b_i - c_i\left(\frac{b_i}{2c_i}\right)\right) + p_T^0 w_{Si}^1\right] + \sum_{A_i^-}\left[\gamma_{J(i)}^l\left(b_i - c_i\gamma_{J(i)}^l\right) + p_T^0 w_{Si}^1\right] \\
&\quad + \sum_{B_i^+}\left[\left(\gamma_{J(i)}^l - w_{Si}^2 - \frac{b_i}{2c_i}\right)\left(b_i - c_i\left(\gamma_{J(i)}^l - w_{Si}^2 + \frac{b_i}{2c_i}\right)\right) + p_T^0 w_{Si}^2\right] \\
&\quad + \sum_{B_i^- \times \mathcal{W}_l^-} \left((0)(b_i - c_i * 0) + p_T^0 * 0\right) + \sum_{B_i^- \times \mathcal{W}_l^+}\left[(-w_{Si}^2)(b_i - c_i(2\gamma_{J(i)}^l - w_{Si}^2)) + p_T^0 w_{Si}^2\right] \\
&\quad + \sum_{C_i}(0)(b_i - c_i(2\gamma_{J(i)}^l)) + \sum_{D_i}\left[w_{Bi}\left(b_i - c_i(2\gamma_{J(i)}^l + w_{Bi})\right) - p_T w_{Bi}\right] \\
&= \sum_{A_i^+}\left[p_T^0\gamma_{J(i)}^l - \left(\frac{b_i}{2}\right)^2\frac{1}{c_i}\right] + \sum_{A_i^-}\gamma_{J(i)}^l\left[p_T^0 - b_i + c_i\gamma_{J(i)}^l\right] \\
&\quad + \sum_{B_i^+}\left(\left(\frac{b_i}{2c_i} - \frac{p_T^0}{2c_i} - \frac{b_i}{2c_i}\right)\left(b_i - c_i\left(\frac{b_i}{2c_i} - \frac{p_T^0}{2c_i} + \frac{b_i}{2c_i}\right)\right) + p_T^0 w_{Si}^2\right) \\
&\quad + \sum_{B_i^- \times \mathcal{W}_l^+}\left[(-w_{Si}^2)\left(b_i - c_i\left(\gamma_{J(i)}^l + \frac{b_i}{2c_i} - \frac{p_T^0}{2c_i}\right)\right) + p_T^0 w_{Si}^2\right] \\
&\quad + \sum_{D_i}\left[w_{Bi}\left(b_i - c_i(\gamma_{J(i)}^l + \frac{b_i}{2c_i} - \frac{p_T}{2c_i})\right) - p_T w_{Bi}\right] \\
&= \sum_{A_i^+}\left[p_T^0\gamma_{J(i)}^l - \left(\frac{b_i}{2}\right)^2\frac{1}{c_i}\right] + \sum_{A_i^-}\gamma_{J(i)}^l\left[p_T^0 - b_i + c_i\gamma_{J(i)}^l\right] + \sum_{B_i^+}\left[p_T^0 w_{Si}^2 - \left(\frac{p_T^0}{2}\right)^2\left(\frac{1}{c_i}\right)\right] \\
&\quad + \sum_{B_i^- \times \mathcal{W}_l^+}\left[w_{Si}^2\left(\frac{p_T^0}{2} - \frac{b_i}{2} + c_i\gamma_{J(i)}^l\right)\right] + \sum_{D_i}\left[w_{Bi}\left(\frac{b_i}{2} - \frac{p_T}{2} - c_i\gamma_{J(i)}^l\right)\right].
\end{aligned}
\tag{24}
$$